The Electric Vehicle Routing Problem with Energy Consumption Uncertainty

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Abstract

Compared with conventional freight vehicles, electric freight vehicles create less local pollution and are thus generally perceived as a more sustainable means of goods distribution. In urban areas, such vehicles must often perform the entirety of their delivery routes without recharging. However, their energy consumption is subject to a fair amount of uncertainty, which is due to exogenous factors such as the weather and road conditions, endogenous factors such as driver behaviour, and several energy consumption parameters that are difficult to measure precisely. Hence we propose a robust optimization framework to take into account these energy consumption uncertainties in the context of an electric vehicle routing problem. The objective is to determine minimum cost delivery routes capable of providing strong guarantees that a given vehicle will not run out of charge during its route. We formulate the problem as a robust mixed integer linear program and solve small instances to optimality using robust optimization techniques. Furthermore, we develop a two-phase heuristic method based on large neighbourhood search to solve larger instances of the problem, and we conduct several numerical tests to assess the quality of the methodology. The computational experiments illustrate the trade-off between cost and risk, and demonstrate the influence of several parameters on best found solutions. Furthermore, our heuristic identifies 42 new best solutions when tested on instances of the closely related robust capacitated vehicle routing problem.

Keywords: electric vehicle routing, robust optimization, city logistics, metaheuristics

1. Introduction

There currently exists a strong interest on the part of the transportation science community in the design of distribution schemes that may reduce the environmental impact of goods distribution, namely in the context of city logistics. Whereas some researchers have focused on taking emission costs into account when routing conventional vehicles (e.g., Jabali et al. 2012, Bektaş and Laporte 2011), others have dealt with the challenges arising from the use of cleaner alternatives such as electric freight vehicles (EFVs) (Pelletier et al. 2016). Indeed, EFVs create less local air pollution and less noise than conventional vehicles, and are therefore considered to be a more sustainable means of goods distribution. Despite the environmental benefits of EFVs, there exist significant issues associated with their integration into goods distribution schemes, most notably their high initial investments, limited range and payload, long recharging times, and the scarcity of public charging stations. In an urban environment, however, delivery routes tend to be shorter than the range of currently available EFVs (Feng and Figliozzi 2013), which means that some of the aforementioned issues, namely those concerning recharging times and public charging infrastructures, can be averted by recharging the batteries at a central depot. Indeed, companies that use EFVs for goods distribution will usually charge them on company grounds overnight and will rarely use public charging stations (Naberezhnykh et al. 2012, E-Mobility NSR 2013, Quak et al. 2017, Morganti and Browne 2018). This practice also mitigates cargo security concerns and avoids inefficient use of drivers' time during en route charging. Furthermore, depot charging offers the possibility of benefiting from lower energy costs through commercial off-peak electricity rates at the depot (Pelletier et al. 2018).

In the case of exclusive charging EFVs at the depot, vehicle routes must be planned carefully as a result of the limited range of the vehicles, and in a way that ensures that the vehicles will be able to perform their full route with their departing battery charge. This is particularly important in the presence of strong uncertainties surrounding the energy consumption of the vehicles along the planned routes. Indeed, Asamer et al. (2016) show that several parameters of the comprehensive emissions model of Barth et al. (2005) commonly used to estimate energy consumption in electric vehicle routing problems (EVRPs) are difficult to measure or depend on uncontrolable exogenous factors, and it is therefore wiser to think of their value as lying within an uncertainty range. Yi and Bauer (2017) also demonstrate how environmental factors such as wind speed, weather, road surface conditions, and temperature can significantly alter the achievable range of electric vehicles. Bingham et al. (2012) reach similar conclusions concerning the influence of driver behaviour, and note that driving less aggressively (e.g., with fewer acceleration phases) can yield substantial energy consumption savings.

The scientific aim of this paper is to introduce, model and solve a practical transportation problem in which EFVs must be optimally routed with strong guarantees that they never get stranded along their routes regardless of the realization of energy consumption uncertainties. We refer to this problem as the electric vehicle routing problem with energy consumption uncertainty (EVRP-ECU) and solve it by means of a robust optimization framework. With this goal in mind, Section 2 provides a short review of the literature related to goods distribution with electric vehicles and robust vehicle routing problems. Section 3 describes the EVRP-ECU and presents its mathematical formulation. Section 4 details two robust optimization approaches that can be used to solve small instances of the problem to optimality. Section 5 presents a two-phase heuristic method for larger instances in which candidate routes are generated via a first phase based on large neighbourhood search (LNS) and are then assembled via a set partitioning (SP) formulation in the second phase. Section 6 provides the results of extensive computational experiments. Finally, Section 7 presents our conclusions.

2. Related literature

Several new routing problems associated with the use of EFVs have been studied since 2010 in order to ease their integration into distribution operations (e.g., Conrad and Figliozzi 2011, Preis et al. 2014, Schneider et al. 2014, Felipe et al. 2014, Goeke and Schneider 2015, Montoya et al. 2017, Schiffer and Walther 2017, Hiermann et al. 2016, 2019). The generic objective in these problems is to use a fleet of EFVs to deliver goods to a set of customers at minimal cost. A recurring feature of the proposed models is that the vehicles can stop at charging stations along their delivery routes to recharge their batteries if needed, thus distinguishing these problems from conventional vehicle routing problems (VRPs) for which the planning of intraroute refueling stops is rarely required. Indeed, many fuel stations are available for conventional vehicles and refueling times are negligible regardless of the fuel station, whereas for EFVs the recharging infrastructure is scarce and recharging times are longer and can vary significantly depending on the type of recharging station. Although en route recharging is rarely needed in urban delivery routes, such studies remain relevant since they could facilitate the use of EFVs outside urban environments (e.g., for mid- and long-haul goods distribution).

In addition, because of the limited battery capacity and the resulting shorter achievable range of EFVs, such routing problems require solution methods that can accurately monitor the vehicles' energy consumption in order to ensure they never get stranded. Some studies assume that the battery discharges itself at a linear rate with respect to distance (e.g., Felipe et al. 2014, Schneider et al. 2014, Hiermann et al. 2016). Others have considered a more detailed energy consumption model based on the comprehensive emissions model of Barth et al. (2005) which computes the required energy to traverse any given arc based on certain vehicle and arc characteristics (e.g., speed, acceleration, mass, elevation, frontal area, rolling friction, air drag). This energy consumption model was first applied in a VRP setting for the pollution-routing problem to estimate fuel consumption and emissions (Bektaş and Laporte 2011), and subsequently in many EVRP studies to track the state of charge of the vehicles' batteries (e.g., Preis et al. 2014, Goeke and Schneider 2015, Lebeau et al. 2015).

Many existing studies have also considered stochastic variants of the VRP in which the objective is to minimize the total expected cost. The problem inputs that are assumed to be uncertain in such problems are usually customer demands, travel and service times, or customer presence, and these uncertainties are generally handled via a priori optimization or through the reoptimization modeling paradigm (Gendreau et al. 2016). The issue of uncertain energy consumption has not yet been studied in VRP settings due to the ease with which intraroute refueling stops for conventional vehicles can be incorporated in a solution.

Robust optimization is a methodology that has increased in popularity in the last two decades, as witnessed by the growing number of publications on this topic. The main benefit of robust optimization lies in its ability to incorporate uncertainty into a problem in a relatively tractable way. This is typically achieved by defining uncertainty sets containing realizations of the uncertain parameters. Moreover, strong probabilistic guarantees can often be attained with very limited information regarding the underlying distribution of the uncertain parameters, while still achieving less conservative solutions than by working with worst-case values (Ben-Tal et al. 2009).

To the best of our knowledge, only Fontana (2013) has considered the uncertainty surrounding the energy consumption of electric vehicles and has modeled the problem within a robust optimization framework. However, this author considers the idea of uncertainty surrounding the energy consumption of electric vehicles in an optimal path problem setting, while we extend it to a VRP setting in which the vehicle loads influence energy consumption. Schiffer and Walther (2018) have also used a robust optimization framework in an EVRP context, but their work considers the problem of simultaneously routing EFVs and locating charging stations to be used en route in the presence of uncertain customer locations, demand, and service time windows. Examples of other studies on robust VRPs include the work of Sungur et al. (2008), Gounaris et al. (2013) and Gounaris et al. (2016) on the capacitated VRP with demand uncertainty; the work of Agra et al. (2012) and Braaten et al. (2017) on the VRP with time windows and travel time uncertainty; the work of Hu et al. (2018) on the VRP with time windows and both demand and travel time uncertainties; and the work of Lee et al. (2012) on the VRP with deadlines, as well as demand and travel time uncertainties.

In summary, the main contributions of this study are to model and solve a practical problem resulting from the fact that EFVs must be able to perform their full delivery routes in urban areas with their departing battery charge, despite uncertainties surrounding their energy consumption. We believe that this paper constitutes a valuable complement to the existing EVRP studies on the design of routes that incorporate en route recharging. We note that such studies relate to the use of EFVs for mid- and long-haul goods distribution, while our objective is to support the use of EFVs for urban operations specifically. This paper is indeed the first to consider the important issue of uncertain energy consumption in an EVRP setting without en route recharging. We use a robust optimization approach to take into account such energy consumption uncertainties (both exact and heuristic solution methods are proposed), and we conduct extensive numerical tests to demonstrate the quality of our methodology. We therefore believe that this study successfully deals with a key challenge arising from the use of EFVs as a more sustainable means of goods distribution, which should be of interest to both transportation scientists and fleet managers.

3. The electric vehicle routing problem with energy consumption uncertainty

The objective of this section is to formally introduce the EVRP-ECU. The section is organized as follows. We first provide a general description of the EVRP-ECU in Section 3.1. In Section 3.2 we elaborate on how the expected values for the arcs' uncertain energy consumption parameters can be approximated. Section 3.3 provides comments on the incorporation of the uncertainty surrounding those energy consumption parameters. The mathematical formulation of the EVRP-ECU is then given in Section 3.4, and finally we present in Section 3.5 the uncertainty sets that we use in our computational experiments.

3.1 Problem description

The EVRP-ECU is defined on a complete directed graph $G = (N, A)$. The node set N is partitioned into $\{N_0, \{0\}\}$, where N_0 is the set of n customers, and node 0 is the depot. The set of arcs is denoted by A. We denote by K the set of homogeneous EFVs that must be used to visit the customers. Each vehicle has a load capacity L (kg) and a battery capacity Q (kWh). Each node i has a demand q_i (kg), with $q_i > 0$ for $i \in N_0$ and $q_0 = 0$.

Each arc (i, j) is assigned a length d_{ij} (m), an expected empty-vehicle energy consumption a_{ij} (kWh), and an expected load-dependent energy consumption b_{ij} (kWh/kg). We assume that the realized empty-vehicle and load-dependent energy consumption values along the arcs will deviate from the expected ones due to the uncertainty surrounding several parameters in the underlying energy consumption model used to estimate a_{ij} and b_{ij}. This will be explained in more detail in Section 3.3. A fixed cost of c_F is incurred for dispatching a vehicle, representing the daily salary of a driver. Parameters c_E and c_M represent energy (\$/kWh) and maintenance (\$/m) costs, respectively.

The following decision variables are required for the formulation of the EVRP-ECU. Binary variables x_{ijk} take value 1 if and only if vehicle k travels on arc (i, j). Variables f_{ijk} refer to the load (kg) carried on arc (i, j) by vehicle k. **Variables w_k refer to the worst-case energy consumption (kWh) that could occur for vehicle k out of all energy consumption realizations in the uncertainty set U_k (which will be formally defined in Section 3.3). We assume that the uncertainty sets U_k are calibrated so as to ensure a probability of at most β that the realized energy consumption of vehicle k will be larger than w_k (with β chosen by the decision maker). For example, if parameter β is set to 0.01 and variable w_k is assigned a value of 40 kWh in an optimal solution to the EVRP-ECU for some vehicle k, then the probability that the delivery route of vehicle k will consume more than 40 kWh will be at most 1%. The objective of the EVRP-ECU is to**

determine a set of routes that minimizes the sum of fixed, maintenance and worst-case energy costs. Furthermore, each customer demand is satisfied by exactly one vehicle, routes start and end at the depot, and total route loads are at most L. Finally, the worst-case energy consumption w_k of each vehicle is at most Q (i.e., the probability that vehicle k can complete its route without getting stranded is at least $1-\beta$).

3.2 Estimating the expected values of the arcs' uncertain energy consumption parameters

The expected values a_{ij} and b_{ij} for the arcs' energy consumption parameters can be approximated using a two-phase approach similar to those discussed by Goeke and Schneider (2015) and Basso et al. (2019). Acknowledging that an arc (i,j) in a VRP graph typically represents a precomputed shortest path between nodes i and j on a more detailed road graph, the idea is to first compute such shortest paths in the road graph, and then determine values representing the expected empty-vehicle and load-dependent energy consumption of a vehicle traveling that path. First assume that each vehicle in K has a curb mass w (kg), a frontal area A (m^2), an air drag coefficient C_d, and an auxiliary power demand P (W). Let $R = (N_R, A_R)$ be a directed graph representing the underlying road graph in which the shortest paths representing the arcs in G are computed. The nodes in set N_R can be assumed to be road intersections, and the arcs in set A_R can be assumed to be road segments. Then if a road segment $(l,m) \in A_R$ (i.e., a road section between intersections $l \in N_R$ and $m \in N_R$) is traveled at constant speed v_{lm} (m/s) by a vehicle in K, then the amount of mechanical energy e_{lm} (kWh) required for that vehicle to travel that road segment with a load f is computed with the longitudinal dynamics model from Asamer et al. (2016), which estimates energy consumption in a similar way to the comprehensive emissions model of Barth et al. (2005):

$$e_{lm} = \frac{1}{3.6 \cdot 10^6}\left((w+f)gd_{lm}\sin\theta_{lm} + (w+f)gC_r d_{lm}\cos\theta_{lm} + 0.5AC_d\rho v_{lm}^2 d_{lm}\right), \tag{1}$$

where g is the gravitational constant, ρ is the air density (kg/m^3), C_r is the rolling friction coefficient, and d_{lm} and θ_{lm} are the length (m) and road angle (rad) of the road segment (l,m). With a travel time of $t_{lm} = v_{lm}/d_{lm}$ associated with road segment (l,m) and assuming the vehicles have a total drivetrain efficiency of ϕ, the amount of energy B_{lm} (kWh) required from the battery on road segment (l,m) is then

$$B_{lm} = \begin{cases} \phi \cdot e_{lm} + \frac{1}{3.6 \cdot 10^6} \cdot P \cdot t_{lm}, & \text{if } e_{lm} \geq 0 \\ \frac{1}{3.6 \cdot 10^6} \cdot P \cdot t_{lm} & \text{if } e_{lm} < 0. \end{cases} \tag{2}$$

Although electric vehicles can sometimes recharge their battery when braking and driving downhill by using the electric motor as a generator, for sake of simplicity we do not consider energy recuperation in this study. This also makes the EVRP-ECU solution even more protected against energy consumption uncertainties (i.e., if the vehicle ends up occasionally recuperating some energy while performing its route, the solution becomes even more robust).

Note that acceleration can also be incorporated into the estimation of e_{lm} if necessary. We first show how this can be done for the simple case of constant acceleration throughout an entire road segment, and we then illustrate how this simple case can be used to establish more involved structures of e_{lm}. If road segment (l, m) is traveled by starting at time $t = 0$ at an initial speed of v_I and accelerating or decelerating at a constant acceleration a during the entire road segment, then a speed of

$$v_{lm} = \sqrt{v_I^2 + 2ad_{lm}} \tag{3}$$

will be reached at the end of the road segment at time

$$t_{lm} = \frac{2d_{lm}}{v_I + v_{lm}}. \tag{4}$$

In this case, the amount of mechanical energy e_{lm} (kWh) required for vehicle $k \in K$ to travel that road segment with a load of f becomes

$$e_{lm} = \frac{1}{3.6 \cdot 10^6}\left((w+f)\alpha a d_{lm} + (w+f)gd_{lm}\sin\theta_{lm} + (w+f)gC_r d_{lm}\cos\theta_{lm} + \int_0^{t_{lm}} 0.5AC_d\rho(v_I + at)^3 dt\right), \tag{5}$$

where α is a mass factor greater than one to account for the rotational inertia of the vehicle's moving parts (Asamer et al. 2016). The integral in the last term can be developed to yield

$$e_{lm} = \frac{1}{3.6 \cdot 10^6}\left((w+f)\alpha a d_{lm} + (w+f)gd_{lm}\sin\theta_{lm} + (w+f)gC_r d_{lm}\cos\theta_{lm} + 0.5AC_d\rho d_{lm}(v_I^2 + \frac{v_{lm}^2 - v_I^2}{2})\right). \tag{6}$$

Equations (4)–(6) can then be used in (2) to compute B_{lm}. Finally, if road segment (l, m) is traveled by starting at an initial speed v_I at time $t = 0$, accelerating at a constant rate a^+ until reaching a speed of v_{lm}, traveling at a constant speed v_{lm} for a certain period of time and then decelerating at a constant rate a^- to reach a speed v_F at time $t = t_{lm}$, then the distance and time traveled in each phase (i.e., acceleration, constant speed, and deceleration) can be computed through equations (3) and (4), respectively. Equations (1) and (6) can then be used to determine the required mechanical energy during each phase. Each of these values can subsequently be used in (2) to compute the amount of energy required by the battery during each phase.

Regardless of the case, the final equation for B_{lm} will always be linear in f. We derive a_{lm} and b_{lm} so that $B_{lm} = a_{lm} + b_{lm} \cdot f$. Thus, once a shortest path S_{ij} has been found for each arc $(i, j) \in A$, we can aggregate the road segment energy consumption parameters of each path to determine the net expected values for the empty-vehicle and load-dependent energy consumption of the fleet vehicles when traveling arc (i, j):

$$a_{ij} = \sum_{(l,m) \in S_{ij}} a_{lm} \quad \forall (i,j) \in A \tag{7}$$

$$b_{ij} = \sum_{(l,m) \in S_{ij}} b_{lm} \quad \forall (i,j) \in A. \tag{8}$$

3.3 Incorporating uncertainty

An issue associated with the values a_{ij} and b_{ij} describing the expected energy consumption on arc (i,j) is of course that several uncertain parameters are needed to estimate them. As discussed by Asamer et al. (2016) and Yi and Bauer (2017), many parameters in the energy consumption model used in the previous section may vary according to some operating conditions, or are difficult to measure precisely. For example, the rolling friction coefficient C_r is difficult to measure and depends on the road surface, the tires, the travel speed, and even the temperature. The air drag coefficient C_d may vary depending on vehicle shape, opened windows, wind speed, and the vehicle's angle of attack. The air density ρ depends on factors such as temperature, atmospheric pressure, and humidity. The drivetrain efficiency ϕ can vary with speed and power. The auxiliary power P depends on driver behaviour and environmental conditions (air conditioning, heating, light, radio). Moreover, driver behaviour and external factors such as traffic conditions may influence the vehicle's travel speed and acceleration cycle. Therefore, even if we compute a_{ij} and b_{ij} with expected values for all these uncertain parameters, it is likely that the realized empty-vehicle and load-dependent energy consumption along some arcs will deviate from a_{ij} and b_{ij} for each vehicle.

To deal with these uncertainties, we derive a robust optimization model by integrating uncertainty on how much the realized energy consumption of a given vehicle k on each arc in A will deviate from its expected value through uncertainty set U_k. Hence we assume that the maximum deviations $\hat{a}_{ij} \geq 0$ and $\hat{b}_{ij} \geq 0$ from a_{ij} and b_{ij}, respectively, are known. For example, the methodology described in Section 3.2 could be applied with worst-case values for the uncertain parameters mentioned in the previous paragraph to estimate $\hat{a}_{ij}$ and $\hat{b}_{ij}$. Asamer et al. (2016) report worst-case values for some of the uncertain parameters in the energy consumption model (e.g., C_r, C_d, ρ, ϕ, P), based on the literature and on measurements from an electric vehicle. Uncertainty ranges for speed and acceleration can then be estimated on the basis of the road graph and of the driver's behaviour, which can further be used with worst-case values of C_r, C_d, ρ, ϕ, and P to determine $\hat{a}_{ij}$ and $\hat{b}_{ij}$. Of course, historical data can also be used to determine $\hat{a}_{ij}$ and $\hat{b}_{ij}$ when available.

Let $Z_k \in \mathbb{R}^{|A|}$ be a random vector with known support $[-1,1]$ representing the deviation of vehicle k's empty-vehicle and load-dependent energy consumption along each arc from their expected values. Once we pick a solution to the EVRP-ECU (i.e., in terms of the variables x_{ijk}, f_{ijk} and w_k), we assume an adversary can select any $\zeta_k \in U_k$ to make vehicle k's route energy-infeasible, where ζ_k is a realization of Z_k. A given element $\zeta_k \in U_k$ thus provides a ζ_{ijk} value that sets the realized empty-vehicle and load-dependent energy consumption parameters for vehicle k along arc (i, j) as $a_{ij} + \hat{a}_{ij} \cdot \zeta_{ijk}$ and $b_{ij} + \hat{b}_{ij} \cdot \zeta_{ijk}$, respectively. The idea is to choose and calibrate sets U_k so as to achieve the desired protection level β by covering enough realizations of the random vectors Z_k. We will use sets U_k to approximate the following chance constraint for each vehicle in the mathematical formulation of Section 3.4:

$$P\left(\sum_{(i,j)\in A}\Big(a_{ij}\cdot x_{ijk} + b_{ij}\cdot f_{ijk} + (\hat{a}_{ij}\cdot x_{ijk} + \hat{b}_{ij}\cdot f_{ijk})\cdot Z_{ijk}\Big) \leq w_k\right) \geq 1-\beta. \tag{9}$$

We assume that the expected values a_{ij} and b_{ij} as well as the maximum deviations $\hat{a}_{ij}$ and $\hat{b}_{ij}$ are the same for each vehicle for a given arc (i, j), and hence we do not index them by vehicle.

3.4 Mathematical formulation

The EVRP-ECU is formulated as the following robust mixed integer optimization model:

$$\text{minimize} \quad \sum_{k\in K}\sum_{j\in N_0} c_F\cdot x_{0jk} + \sum_{k\in K}\sum_{(i,j)\in A} c_M\cdot d_{ij}\cdot x_{ijk} + \sum_{k\in K} c_E\cdot w_k \tag{10}$$

subject to

$$\sum_{j\in N_0} x_{0jk} \leq 1 \qquad k\in K \tag{11}$$

$$\sum_{k\in K}\sum_{j\in N\setminus\{i\}} x_{ijk} = 1 \qquad i\in N_0 \tag{12}$$

$$\sum_{j\in N\setminus\{i\}} x_{jik} = \sum_{j\in N\setminus\{i\}} x_{ijk} \qquad i\in N, k\in K \tag{13}$$

$$q_j\cdot x_{ijk} \leq f_{ijk} \leq (L-q_i)\cdot x_{ijk} \qquad (i,j)\in A, k\in K \tag{14}$$

$$\sum_{k\in K}\sum_{j\in N\setminus\{i\}} f_{jik} - \sum_{k\in K}\sum_{j\in N\setminus\{i\}} f_{ijk} = q_i \qquad i\in N_0 \tag{15}$$

$$\sum_{(i,j)\in A}\Big((a_{ij}+\hat{a}_{ij}\cdot\zeta_{ijk})\cdot x_{ijk} + (b_{ij}+\hat{b}_{ij}\cdot\zeta_{ijk})\cdot f_{ijk}\Big) \leq w_k \qquad k\in K, \zeta_k\in U_k \tag{16}$$

$$0\leq w_k\leq Q \qquad k\in K \tag{17}$$

$$x_{ijk}\in\{0,1\} \qquad (i,j)\in A, k\in K. \tag{18}$$

The objective function (10) minimizes total fixed, maintenance and worst-case energy costs.

Constraints (11) ensure that each vehicle is dispatched at most once. Constraints (12) force each customer node to be visited exactly once. Constraints (13) are flow conservation equations. Constraints (14) ensure that the load capacity is respected, that a load can only be carried along an arc if a vehicle travels that arc, and bound that load appropriately for each arc. Constraints (15) state that the incoming load minus the outgoing load for each customer node must be equal to its demand. These constraints also serve as subtour elimination constraints. Constraints (16) are used to determine the worst-case total energy consumption w_k that could occur for vehicle k out of all possible scenarios in the uncertainty set U_k, which is assumed to have been calibrated to approximate the chance constraint (9). Constraints (17) ensure that the worst-case energy consumption of each vehicle is at most the battery capacity Q. Finally, constraints (18) define the domain of the x_{ijk} variables.

Note that once the problem is solved, even if the optimal values for the w_k variables are much smaller than Q, the information they provide can still be useful from both a depot scheduling and a battery degradation perspective. Indeed, frequently charging a battery to a high level (and keeping it at this level for a lenghty period) can shorten its expected lifespan (see, e.g., Lunz et al. 2012). Hence companies using EFVs should ideally try to charge the minimum required for their operations, and do this as closely as possible to the vehicles' departure times (Pelletier et al. 2017). This is especially true considering that having to replace the battery of EFVs over the course of their lifetime has been shown to significantly impact their business case (Davis and Figliozzi 2013, Feng and Figliozzi 2013, Lee et al. 2013). The EVRP-ECU can thus provide valuable information regarding the smallest "safe" amount of energy to recharge in each vehicle. Moreover, when charging is only performed at the depot, full charges may not be possible nor optimal because of time-dependent energy costs, demand charges, battery degradation, depot charging infrastructures or multi-shift operational contexts (Pelletier et al. 2018). The information given by the solution to the EVRP-ECU can thus also help plan the depot charging activities by knowing the smallest departing battery level w_k required for each vehicle.

3.5 Uncertainty sets

As mentioned in Section 3.3, the uncertainty sets U_k used in constraints (16) are assumed to be designed so as to ensure a probability at most equal to β that the real energy consumption of vehicle k be larger than w_k, i.e., we want to approximate chance constraint (9) for each vehicle through constraints (16). It is ultimately up to the decision maker to choose and calibrate the uncertainty sets based on what is known about the random vectors Z_k so as to reach the desired trade-off between cost and risk. In what follows we present five uncertainty sets to illustrate how different alternatives can be considered depending on the desired protection level and on the assumptions made about the distribution of the uncertain parameters. For example, in order to have a zero probability that the real energy consumption of vehicle k will be larger than w_k, one could use the entire support of random vector Z_k through the following box uncertainty set (Ben-Tal et al. 2009):

$$U_k^1 = \{\zeta_k \in \mathbb{R}^{|A|} \mid -1 \leq \zeta_{ijk} \leq 1 \ \forall (i,j) \in A\}, \tag{19}$$

which is equivalent to solving the problem with the worst-case energy parameter values $(a_{ij} + \hat{a}_{ij})$ and $(b_{ij} + \hat{b}_{ij})$ for each arc. However, one does not expect the worst-case value to occur on every arc simultaneously, and cost savings may be achievable by being less conservative but still ensuring that the real energy consumption of vehicle k is almost never larger than w_k. Budgeted uncertainty sets calibrated through parameter Γ could therefore be used (Ben-Tal et al. 2009):

$$U_k^2 = \{\zeta_k \in \mathbb{R}^{|A|} \mid -1 \leq \zeta_{ijk} \leq 1 \ \forall (i,j) \in A, \sum_{(i,j)\in A} |\zeta_{ijk}| \leq \Gamma\}, \tag{20}$$

thereby only allowing up to Γ arcs to deviate from their expected energy consumption values on each route in order to render that route energy-infeasible. There also exist studies (e.g., Poss 2013) that show how allowing the size of the uncertainty set to depend on the number of non-zero decisions in combinatorial problems can often yield less conservative solutions while providing the same probabilistic guarantees. Hence one may consider route-dependent budgeted uncertainty sets calibrated with parameters θ_0 and θ:

$$U_k^3 = \{\zeta_k \in \mathbb{R}^{|A|} \mid -1 \leq \zeta_{ijk} \leq 1 \ \forall (i,j) \in A, \sum_{(i,j)\in A} |\zeta_{ijk}| \leq \theta_0 + \theta \sum_{(i,j)\in A} x_{ijk}\} \quad \forall k \in K, \tag{21}$$

where the budget of uncertainty that can affect vehicle k now depends on the length of the route. Hu et al. (2018) have proposed similar route-dependent uncertainty sets. Another option to prevent all parameters from taking their worst-case value simultaneously may be to use the known support of Z_k intersected with a ball of radius γ centered at the origin (Ben-Tal et al. 2009):

$$U_k^4 = \{\zeta_k \in \mathbb{R}^{|A|} \mid -1 \leq \zeta_{ijk} \leq 1 \ \forall (i,j) \in A, ||\zeta_k||_2 \leq \gamma\}. \tag{22}$$

If all that is known about Z_k is that it is symmetrically distributed on the interval $[-1, 1]$ and that its components are independent, then any of the sets U_k^2, U_k^3 or U_k^4 can be calibrated to approximate (9) (see, e.g., Ben-Tal et al. 2009, Poss 2013). There also exist some uncertainty sets that can be used to account for the correlation between the components of Z_k, which may be relevant for the EVRP-ECU considering that driver behaviour and external conditions like the weather and road conditions should not differ too much along a given route. For example, ellipsoidal uncertainty sets defined by the following can be useful if all that is known about Z_k is that it is distributed with a mean of zero and a covariance matrix $\sum \in \mathbb{R}^{|A|\times|A|}$:

$$U_k^5 = \{\zeta_k \in \mathbb{R}^{|A|} \mid \sqrt{\zeta_k^T \sum{}^{-1} \zeta_k} \leq r\}, \tag{23}$$

where the parameter r determines the cardinality of the set and can hence be chosen according to the desired level of protection and any additional assumptions on the distribution of Z_k, e.g., normal or arbitrary (El Ghaoui et al. 2003). A few other uncertainty sets that can be used to deal with the correlation among the components of Z_k can be found in Bandi and Bertsimas (2012), Yuan et al. (2016), Jalilvand-Nejad et al. (2016) and Gounaris et al. (2013, 2016). Some ideas on how to construct uncertainty sets in the presence of historical data are also discussed in Bertsimas et al. (2018). In Section 6, we conduct numerical tests with each of the above uncertainty sets except U_k^2, since it can be viewed as a special case of U_k^3 with $\theta = 0$.

4. Exact methods

Non-discrete uncertainty sets such as those presented in Section 3.5 amount to having an infinite number of constraints (16) in the robust optimization model, thus rendering the model intractable for commercial solvers, even for small instances. We have therefore implemented robust optimization techniques in order to solve small instances of the problem to optimality. These include 1) reformulating the model of Section 3.4 so as to obtain a tractable deterministic mixed integer linear program in the case of polyhedral uncertainty sets, and 2) using a cutting-plane method. In what follows we briefly present these methods. Their results will be used to evaluate the performance of the metaheuristic presented in Section 5.

4.1 Reformulation

If the uncertainty set is polyhedral, then the model of Section 3.4 can be reformulated to yield a tractable deterministic mixed integer linear program. We illustrate this with the budgeted uncertainty sets U_k^2 and U_k^3. First, because we assume that $\hat{a}_{ij}, \hat{b}_{ij} \geq 0 \;\; \forall (i,j) \in A$, it is never beneficial for the adversary to set $\zeta_{ijk} < 0$ for an arc. Hence we can reformulate sets U_k^2 and U_k^3 by setting $0 \leq \zeta_{ijk} \leq 1$ instead of $-1 \leq \zeta_{ijk} \leq 1$, and by dropping the absolute value in the sums.

In the case of the route-independent budgeted sets U_k^2, for a given solution of the EVRP-ECU (i.e., for fixed variables) we can check whether constraints (16) are satisfied for vehicle k by verifying whether the optimal solution value of the following linear program is at most $w_k - \sum_{(i,j)\in A} \big(a_{ij} \cdot x_{ijk} + b_{ij} \cdot f_{ijk}\big)$:

$$\text{maximize} \quad \sum_{(i,j)\in A} \big(\hat{a}_{ij} \cdot x_{ijk} + \hat{b}_{ij} \cdot f_{ijk}\big) \cdot \zeta_{ijk} \tag{24}$$

subject to

$$\sum_{(i,j)\in A} \zeta_{ijk} \leq \Gamma \tag{25}$$

$$0 \leq \zeta_{ijk} \leq 1 \qquad (i,j) \in A. \tag{26}$$

The dual of the above linear program can be written with variables λ_k and σ_{ijk} as

$$\text{minimize} \quad \Gamma \cdot \lambda_k + \sum_{(i,j)\in A} \sigma_{ijk} \tag{27}$$

subject to

$$\lambda_k + \sigma_{ijk} \geq \hat{a}_{ij} \cdot x_{ijk} + \hat{b}_{ij} \cdot f_{ijk} \qquad (i,j) \in A \tag{28}$$

$$\lambda_k \geq 0 \tag{29}$$

$$\sigma_{ijk} \geq 0 \qquad (i,j) \in A. \tag{30}$$

Hence we obtain a tractable deterministic mixed integer linear programming formulation of the EVRP-ECU by replacing constraints (16) in the model of Section 3.4 with

$$\sum_{(i,j)\in A} \big(a_{ij} \cdot x_{ijk} + b_{ij} \cdot f_{ijk} + \sigma_{ijk}\big) + \Gamma \cdot \lambda_k \leq w_k \qquad k \in K, \tag{31}$$

and by adding constraints (28)–(30) for each vehicle $k \in K$, since duality theory ensures that $\Gamma \cdot \lambda_k + \sum_{(i,j)\in A} \sigma_{ijk} \geq \max_{\zeta_k \in U_k} \left\{ \sum_{(i,j)\in A} \big(\hat{a}_{ij} \cdot x_{ijk} + \hat{b}_{ij} \cdot f_{ijk}\big) \cdot \zeta_{ijk} \right\}$ for vehicle k as long as constraints (28)–(30) are satisfied. Note that λ_k and σ_{ijk} are variables in the reformulation.

In the case of the route-dependent budgeted uncertainty sets U_k^3, we would instead need to solve the following to check if constraints (16) are satisfied for k in a given solution:

$$\text{maximize} \quad \sum_{(i,j)\in A} \big(\hat{a}_{ij} \cdot x_{ijk} + \hat{b}_{ij} \cdot f_{ijk}\big) \cdot \zeta_{ijk} \tag{32}$$

subject to

$$\sum_{(i,j)\in A} \zeta_{ijk} \leq \theta_0 + \theta \sum_{(i,j)\in A} x_{ijk} \tag{33}$$

$$0 \leq \zeta_{ijk} \leq 1 \qquad (i,j) \in A. \tag{34}$$

The dual of the above linear program can once again be written with variables λ_k and σ_{ijk}:

$$\text{minimize} \quad \theta_0 \cdot \lambda_k + \sum_{(i,j)\in A} \Big(\theta \cdot x_{ijk} \cdot \lambda_k + \sigma_{ijk}\Big)$$

subject to (28)–(30).

Constraints (16) are therefore equivalent to

$$\sum_{(i,j)\in A} \Big((a_{ij} + \theta \cdot \lambda_k) \cdot x_{ijk} + b_{ij} \cdot f_{ijk} + \sigma_{ijk}\Big) + \theta_0 \cdot \lambda_k \leq w_k \qquad k \in K \tag{35}$$

$$\lambda_k + \sigma_{ijk} \geq \hat{a}_{ij} \cdot x_{ijk} + \hat{b}_{ij} \cdot f_{ijk} \qquad k \in K, (i,j) \in A \tag{36}$$

$$\lambda_k \geq 0 \qquad k \in K \tag{37}$$

$$\sigma_{ijk} \geq 0 \qquad k \in K, (i,j) \in A. \tag{38}$$

Since the x_{ijk} variables are binary, as in Poss (2013) we linearize constraints (35) by introducing real variables u_{ijk} to represent the product of variables x_{ijk} and λ_k. Constraints (35) can thus be replaced with

$$\sum_{(i,j)\in A}\left(a_{ij}\cdot x_{ijk}+\theta\cdot u_{ijk}+b_{ij}\cdot f_{ijk}+\sigma_{ijk}\right)+\theta_0\cdot\lambda_k\leq w_k \quad k\in K \tag{39}$$

$$u_{ijk}\geq\lambda_k-M(1-x_{ijk}) \quad k\in K,(i,j)\in A \tag{40}$$

$$u_{ijk}\geq 0 \quad k\in K,(i,j)\in A, \tag{41}$$

where M is a large constant. We can therefore obtain a tractable deterministic mixed integer linear programming formulation of the EVRP-ECU with route-dependent budgeted uncertainty sets as well. The robust problem can also be reformulated through duality theory as a deterministic problem in the presence of non-polyhedral uncertainty sets like those defined by U_k^4 and U_k^5, but in this case the reformulation gives rise to a second-order cone problem. We therefore present a cutting plane method for these cases in the next section.

4.2 Cutting-plane method

An alternative to reformulating the robust optimization model is to initiate a branch-and-cut procedure on the nominal version of the problem (i.e., without uncertainty), and whenever an integer feasible solution is found, to verify the worst-case energy consumption of each route and add cuts accordingly (Bertsimas et al. 2016). To this end, we need to solve the following problem for each route k in the current integer feasible solution of the search tree:

$$\text{maximize} \quad \sum_{(i,j)\in A}\left(a_{ij}\cdot x_{ijk}+b_{ij}\cdot f_{ijk}+\left(\hat{a}_{ij}\cdot x_{ijk}+\hat{b}_{ij}\cdot f_{ijk}\right)\cdot\zeta_{ijk}\right) \tag{42}$$

subject to

$$\zeta_k\in U_k, \tag{43}$$

where U_k is the considered uncertainty set. Let H_k be the value of the optimal solution of (42)–(43) for the route of vehicle k, achieved with scenario $\overline{\zeta}_k\in U_k$. If 1) the uncertainty sets used are not route-dependent, 2) H_k is larger than the value of variable w_k in the current solution, and 3) the uncertainty sets are the same for all vehicles, then we add the following cuts:

$$\sum_{(i,j)\in A}\left((a_{ij}+\hat{a}_{ij}\cdot\overline{\zeta}_{ijk})\cdot x_{ijl}+(b_{ij}+\hat{b}_{ij}\cdot\overline{\zeta}_{ijk})\cdot f_{ijl}\right)\leq w_l \quad l\in K, \tag{44}$$

since the realization $\overline{\zeta}_k$ is valid for any other route that could be assigned to any vehicle. Constraints (44) force a sufficient worst-case energy consumption of H_k whenever a vehicle performs the route that vehicle k was performing in the current solution, thereby simultaneously forbidding the route whenever $H_k > Q$, due to constraints (17). If the branch-and-cut procedure eventually finds an optimal solution in which all routes are robust, only a subset of constraints (16) would have been added. The solution is therefore also optimal for the entire problem.

However, if the uncertainty set is route-dependent, we cannot add constraints (44) since realization $\overline{\zeta}_k$ it is not necessarily valid for other route lengths. Alternatively, we can add cuts similar to those proposed by Laporte and Louveaux (1993) for the integer L-shaped method. Letting R_k be the set containing the arcs in the route of vehicle k and assuming once again that the uncertainty sets are the same for each vehicle, the following cuts are added if H_k is larger than the value of variable w_k in the current solution, and the uncertainty sets are route-dependent:

$$H_k \cdot \left(\sum_{(i,j)\in R_k} x_{ijl} - \sum_{(i,j)\notin R_k} x_{ijl} \right) - H_k \cdot (|R_k| - 1) \le w_l \quad l \in K. \tag{45}$$

If $H_k \le Q$, then constraints (45) will ensure that the worst-case energy consumption of the vehicle performing route R_k be at least H_k. If $H_k > Q$, then constraints (45) will ensure that no vehicle performs route R_k due to constraints (17). Note that although this is not optimal, some feasible solutions encountered during the branch-and-cut procedure could have vehicles returning to the depot carrying loads. Therefore, one should update the load carried along the route so as to have no returning loads before solving (42)–(43) to obtain H_k and subsequently adding constraints (45). If not, constraints (45) could forbid a route that would otherwise be feasible by making this correction. Finally, we mention that the method discussed here for route-dependent uncertainty sets is also valid for those sets that are not, and that if the uncertainty sets are the same for each vehicle, then (42)–(43) only needs to be solved the first time a specific route is encountered in the search tree.

5. Two-phase metaheuristic

In order to solve large instances of the EVRP-ECU, we propose a two-phase heuristic in which a pool of candidate routes is generated in a first phase, and then combined via an SP formulation in the second phase. Similar two-phase methods have been shown to be quite successful in both deterministic and stochastic routing problem settings (see, e.g., Mendoza and Villegas 2013, Montoya et al. 2016, Mendoza et al. 2016, Montoya et al. 2017). The first phase uses the LNS framework first proposed by Shaw (1998) to construct a pool of routes, thereby destroying and repairing large portions of the solution at each iteration to move from one area of the search space to another. Rather than forcing solutions to remain energy-feasible during the first phase of the procedure with respect to their routes' worst-case energy consumption, we only ensure energy feasibility with respect to their routes' expected energy consumption. However, after destroying and repairing the current solution at each iteration of the LNS in the first phase of the method (and occasionally during and after an additional local search phase), we check whether the resulting solution contains any new robust routes, i.e., routes with a worst-case energy consumption that is less than the battery capacity. If so, we add them to the pool of robust routes. In the second phase, after a number of iterations of the LNS, we solve a set partitioning problem over this pool to generate a complete robust solution. This allows 1) to only have to evaluate the robustness of routes once moves have been implemented, rather then when evaluating a potential move (which could significantly increase the solution times), 2) to generate infeasible solutions during the search in terms of energy, which can act as a diversification mechanism, and 3) to easily adapt the implementation of the algorithm when solving the problem with new uncertainty sets.

In contrast to earlier LNS methods that used only one operator to remove customers and one operator to insert them, we apply several removal and insertion operators, as in the adaptive LNS (ALNS) introduced by Ropke and Pisinger (2006). ALNS is an extension of the LNS framework in which several destroy and repair operators compete against each other to modify the current solution. The probabilities of selecting the different operators are dynamically updated through an adaptive mechanism, thereby allowing operators that perform well on a given instance to be used more frequently. The LNS heuristic we have implemented in the first phase of our heuristic contains all the ingredients of an ALNS procedure besides the adaptive layer. We initially implemented our method within the ALNS framework, but we observed that the adaptive component did not improve solution quality significantly. We therefore opted to keep the probability of selecting each operator fixed throughout the search in order to reduce the number of parameters of the algorithm.

The main steps of our solution method are as follows. We first build an initial solution with a simple constructive heuristic (Section 5.1). Then, at each iteration, random numbers are drawn to determine which removal and insertion operators to apply. Most of our operators are inspired by those used in Ropke and Pisinger (2006), Demir et al. (2012), and Goeke and Schneider (2015). The selected removal operator is then applied to remove a random number of customers n_r between $\underline{n}_c$ and $\overline{n}_c$, or to eliminate an entire route (Section 5.2). The removed customers are then placed in a removal list until they are reinserted in the solution (Section 5.3). A local search (LS) phase is then performed when the repaired solution seems promising (Section 5.4), and the resulting solution is then accepted or rejected according to the simulated annealing mechanism (Section 5.5). Solutions are checked for new robust routes before, after, and occasionally during the LS phase. It is therefore important to efficiently compute the worst-case energy consumption of new routes (Section 5.6). We also update the best found (if any) robust solution during the LNS phase when a solution only contains robust routes. Finally, a robust LS is performed on the best found nominal solution (which is not necessarily robust) throughout the procedure prior to the set partitioning phase, and on the robust solution obtained after solving the set partitioning problem (Section 5.7). The best candidate between the resulting solution and the best found robust solution over the course of the LNS phase is then chosen as the final solution. An outline of the method is provided in Algorithm 1.

Algorithm 1 Two-phase heuristic method

Input: parameters ω, δ, τ, r, and κ
Output: a robust solution X_{Final}

Generate an initial solution $X_{Initial}$ with the savings algorithm
Initiate temperature T according to $X_{Initial}$ and ω
Check robustness of routes in $X_{Initial}$ and add all robust routes to Ω_R
if All routes in $X_{Initial}$ are robust **then**
    $X_{BestRobust} \leftarrow X_{Initial}$
$X_{Cur} \leftarrow X_{Initial}$, $X_{BestNominal} \leftarrow X_{Initial}$
repeat
    Select a removal operator and an insertion operator and apply them on X_{Cur} to get X_{Temp}
    Check robustness of any new routes in X_{Temp} and add all new robust routes to Ω_R
    if All routes in X_{Temp} are robust and $C^R(X_{temp}) < C^R(X_{BestRobust})$ **then**
        $X_{BestRobust} \leftarrow X_{Temp}$
    if $C(X_{Temp}) \leq \delta \cdot C(X_{BestNominal})$ **then**
        if $C(X_{Temp}) \leq \tau \cdot C(X_{BestNominal})$ **then**
            Perform local search on X_{Temp} to get X_{New}, add all new robust routes encountered during the search to Ω_R, and update $X_{BestRobust}$ during the search
        else
            Perform local search on X_{Temp} to get X_{New} and add all new robust routes in X_{New} to Ω_R
            if All routes in X_{New} are robust and $C^R(X_{New}) < C^R(X_{BestRobust})$ **then**
                $X_{BestRobust} \leftarrow X_{New}$
    else
        $X_{New} \leftarrow X_{Temp}$
    if $C(X_{New}) < C(X_{BestNominal})$ **then**
        $X_{Cur} \leftarrow X_{New}$
        $X_{BestNominal} \leftarrow X_{New}$
    else
        if $C(X_{New}) < C(X_{Cur})$ or the *Random insertion* was applied **then**
            $X_{Cur} \leftarrow X_{New}$
        else
            Generate a random number $p \in [0, 1]$
            if $p \leq \exp[-(C(X_{New}) - C(X_{Cur}))/T]$ **then**
                $X_{Cur} \leftarrow X_{New}$
    $T \leftarrow r \cdot T$
until κ iterations have been performed
Perform robust local search on $X_{BestNominal}$ to get $X_{LNS/RLS}$ and add any new robust routes encountered during the search to Ω_R
if All routes in $X_{LNS/RLS}$ are robust and $C^R(X_{X_{LNS/RLS}}) < C^R(X_{BestRobust})$ **then**
    $X_{BestRobust} \leftarrow X_{X_{LNS/RLS}}$
Solve set partioning problem to find a robust solution X_{SP}
Perform robust local search on X_{SP} to get the robust solution $X_{SP/RLS}$
if $X_{BestRobust} \neq \emptyset$ and $C^R(X_{BestRobust}) < C^R(X_{SP/RLS})$ **then**
    $X_{Final} \leftarrow X_{BestRobust}$
else
    $X_{Final} \leftarrow X_{SP/RLS}$

5.1 Initial solution construction

In order to construct an initial solution $X_{Initial}$, we apply the Clarke and Wright savings algorithm (1964). An initial solution is constructed by creating n back-and-forth routes, followed by merging, at each iteration, a route ending at customer i with a route starting at customer j so as to maximize the distance saving $s_{ij} = d_{i0} + d_{0j} - d_{ij}$. We only merge routes when feasibility can be maintained with respect to capacity and expected energy consumption. The initial back-and-forth routes are checked for robustness and added to the pool of robust routes accordingly.

5.2 Removal operators

We now briefly describe the operators that we use in the LNS phase of our method. At the start of each iteration, one of the following removal operators is randomly chosen to remove a certain number of customers from the current solution X_{Cur}.

Random removal randomly selects n_r customers and removes them from the current solution.

Worst removal assigns a value to each customer indicating the difference between the solution value with the customer and without it. These are then stored in an ordered set S and the customer with the $\lfloor |S| \cdot \epsilon^{\lambda_{WR}} \rfloor^{th}$ largest value is removed from the solution (with the 0^{th} referring to the highest value). The parameter ϵ is randomly drawn in $[0, 1)$ according to a continuous uniform distribution and the parameter λ_{WR} is used to randomize the removal. The procedure is reiterated until n_r customers have been removed. Note that when evaluating the difference between the solution value with a customer and without it, the energy costs are computed with the vehicles' expected energy consumption.

Worst energy removal is similar to *Worst removal*, but the cost assigned to a customer j is given by $(a_{ij} + b_{ij} \cdot f_{ijk}) + (a_{jl} + b_{jl} \cdot f_{jlk})$, where i and l are the nodes immediately preceding and following j in its route in the current solution. As with *Worst removal*, a parameter λ_{WE} is introduced to randomize the removal.

Worst distance removal is the same as *Worst energy removal*, but the cost assigned to a customer j is given by $d_{ij} + d_{jl}$, where i and l are the nodes directly preceding and following j in its route in the current solution. The parameter λ_{WD} is introduced to randomize the removal as with the previous two removal operators.

Shaw removal is intended to remove customers that are similar, based on predetermined criteria. First, a customer i is randomly selected and removed from the solution. The relatedness $R(i, j)$ between customer i and a customer j in our implementation is defined as

$$R(i,j) = \mu_1 \cdot \frac{d_{ij}}{\max\limits_{(l,m)\in A}(d_{lm})} + \mu_2 \cdot s_{ij} + \mu_3 \cdot \frac{|q_i - q_j|}{\max\limits_{l\in N_0}(q_l) - \min\limits_{m\in N_0}(q_m)} + \mu_4 \cdot \frac{a_{ij}}{\max\limits_{(l,m)\in A}(a_{lm})},$$

where $s_{ij} = -1$ if i and j are on the same route, and $s_{ij} = 1$ otherwise. The customers in the solution are then stored in S and the customer with the $\lfloor |S| \cdot \epsilon^{\lambda_{Shaw}} \rfloor^{th}$ smallest relatedness value is removed (with the 0^{th} referring to the smallest value). As with the previous three operators, ϵ is randomly selected in $[0, 1)$, and λ_{Shaw} is a parameter used to introduce some randomness in the search process. The procedure is then repeated by setting i as the last removed customer until n_r customers have been removed. **The parameters μ_1, μ_2, μ_3, and μ_4 are used to attribute weights to similarities regarding different (normalized) criteria. Our implementation defines the relatedness $R(i, j)$ between customers i and j based on the distance separating them (weight μ_1), on whether they are in the same route or not (weight μ_2), on the difference between their demands (weight μ_3), and on the expected energy consumption required for an empty vehicle to travel from i to j (weight μ_4).**

Node neighbourhood removal first selects a customer i randomly and removes it from the solution. It then removes the $n_r - 1$ nodes closest to i.

Random route removal randomly selects a route in the solution and removes all its customers.

Distance-based route removal removes the longest route from the current solution.

Energy-based route removal removes the route with the largest expected energy consumption from the current solution.

5.3 Insertion operators

Once the removal phase is completed, one of the following insertion operators is randomly chosen to insert the removed customers back into the partially destroyed solution. The insertion operators only ensure energy feasibility with respect to the vehicles' expected energy consumption, and evaluate energy costs with the vehicles' expected energy consumption when comparing solution values.

Greedy insertion determines the best position where each removed customer can be inserted in the solution, and inserts the customer whose best insertion position yields the smallest increase in the solution value. This process is then reiterated until all removed customers have been inserted. A removed customer i can be inserted in any feasible position in an existing route, or a new route containing only i can be created.

Regret insertion seeks to avoid delaying the insertion of customers that could deteriorate the solution if they are not inserted with a higher priority. A *Regret-k insertion* in our implementation will determine the k best insertion positions of each removed node. Letting c_{ik} refer to the cost of the partial solution resulting from inserting i in its k^{th} best position, *Regret-k insertion* will insert the removed node i yielding the largest value of $\sum_{j=2}^{k} (c_{ij} - c_{i1})$. The LNS phase of our method uses *Regret-2 insertion* and *Regret-3 insertion*.

Randomized greedy insertion is similar to *Greedy insertion*, but the insertion costs associated with the best insertion position of each removed customer are stored in S, and the customer with the $\lfloor |S| \cdot \epsilon^{\lambda_{RG}} \rfloor^{th}$ smallest value is inserted into its best position. As with several of the removal operators, ϵ is randomly selected from $[0, 1)$ and the parameter λ_{RG} is used to randomize the heuristic. The procedure is repeated until all removed customers have been inserted.

Randomized regret-2 insertion is similar to *Regret-2 insertion*, but instead of always inserting the removed customer i with the largest $(c_{i2} - c_{i1})$ value, the selection is randomized as in *Randomized Greedy insertion*, using the parameter λ_{RR} to control the degree of randomization.

Energy-based insertion calculates the cost of inserting a removed customer i along a route, based on the expected energy consumption of the route prior to and after inserting i in it. The operator thus determines the best possible position where each removed node can be inserted in the partial solution based on such energy-based insertion costs, and inserts the customer whose best insertion position results in the smallest increase of the associated route's expected energy consumption.

5.4 Local search

If the solution X_{Temp} resulting from the destroy and repair phase has a nominal cost (i.e., with energy costs computed with the expected energy consumption of vehicles) at most equal to δ times the nominal cost of the best known nominal solution $X_{BestNominal}$, where $\delta > 1$, an LS phase is performed following a first-improvement strategy with respect to nominal costs. We refer to solutions in which energy feasibility and energy costs are evaluated with respect to the vehicles' expected energy consumption as nominal solutions. The LS uses a composite neighbourhood of *2-Opt* (intra-route moves), *2-Opt** (inter-route moves), *Relocate* (intra- and inter-route moves), and *Exchange* (intra- and inter-route moves). Energy feasibility is only ensured with respect to the vehicles' expected energy consumption during the LS.

2-Opt removes two arcs from a route and inserts two other arcs to reconnect the remaining paths, thereby inverting the order in which the nodes between the two removed arcs are visited. We only consider *2-Opt* moves involving at most three nodes between the two removed arcs. *2-Opt** selects two routes and removes an arc from each of them. It then merges the first sequence of each route with the second sequence of the other (Potvin and Rousseau 1995). *Exchange* swaps the positions of two nodes, and *Relocate* shifts one node from its current position to another position.

The neighbourhoods are searched in a way that is similar to that of the LS procedure of Hiermann et al. (2016). The four neighbourhoods are stored in a list that is randomly shuffled prior to each LS phase to determine in which order they will be searched. The first neighbourhood in the list is then searched until the solution cannot be further improved. Once this occurs, the following neighbourhood is explored, and so on. The search restarts with the first neighbourhood in the list once the end of the list is reached. When none of the neighbourhoods can further improve the current solution, the LS phase terminates. The ordering of the potential moves when searching a neighbourhood is also randomly shuffled each time a move is implemented.

5.5 Acceptance criterion

A simulated annealing mechanism is used to determine whether the solution generated at an iteration should be accepted or not. The term $C(X)$ refers to the nominal cost of a solution X, and X_{Cur} and X_{New} refer to the current and new solution at a given iteration, respectively. If $C(X_{New}) < C(X_{Cur})$, then X_{New} is always accepted. If $C(X_{New}) > C(X_{Cur})$, then the solution X_{New} is accepted with probability $\exp[-(C(X_{New}) - C(X_{Cur}))/T]$, where T is the current temperature. We initialize the temperature to a value of T_{init} and multiply it at each iteration by a cooling rate $r \in (0, 1)$. With $X_{Initial}$ referring to the initial solution of the algorithm, T_{Init} is set according to parameter ω, itself determined so that a solution X of value $C(X) = \omega \cdot C(X_{Initial})$ is accepted with probability 0.5. The cooling rate r is set so that the temperature is below 0.001 for the last 20% of the iterations. In Algorithm 1, $C^R(X)$ refers to the robust cost (i.e., with energy costs computed with the worst-case energy consumption of vehicles) of solution X.

Finally, to further diversify the search, when the solution X_{New} is not a new solution for D consecutive iterations of the LNS phase, the nodes that are removed during the following iteration are subsequently inserted back into the solution with a *Random insertion* operator, and the resulting solution is always accepted. The *Random insertion* operator first shuffles the list containing the removed nodes to determine the order in which they will be inserted back into the solution. For each node in the list, a random insertion position is computed iteratively until a feasible insertion position has been found. The less costly option between inserting the node in that position or creating a new route is then implemented.

5.6 Checking robustness

In order to generate a sufficient number of energy-robust routes for the SP phase, we always check whether any new robust routes are in the solution X_{Temp} obtained immediately after applying a removal operator and an insertion operator on X_{Cur}. However, in order to avoid making the set partitioning problem too large and slowing down the LS, we do not always check for new robust routes during the LS when it is performed. When the LS is performed starting from solution X_{Temp}, if the nominal cost of X_{Temp} is less than a given threshold, we check the robustness of any new routes encountered during the LS phase (i.e., whenever we move from the current solution to a neighbour solution during the LS, any new routes in the chosen neighbour solution are checked for robustness and added to the pool of robust routes accordingly). The threshold is set as τ times the nominal cost of the best known nominal solution $X_{BestNominal}$, with τ being a parameter of the algorithm. If the LS is performed starting from solution X_{Temp} but the nominal cost of

X_{Temp} is more than this threshold, then new routes encountered during the LS are not checked for robustness. However, we check the robustness of any new routes in the final solution returned by the LS. Moreover, whenever any solution X is checked for new robust routes during the LNS phase, we also verify whether all its routes happen to be robust. If this is the case, we update the best known robust solution $X_{BestRobust}$ found during the first phase of the heuristic according to whether $C^R(X) < C^R(X_{BestRobust})$ or not. Note that $X_{BestRobust}$ could be empty at the end of the first phase. If it is not, after the SP phase we choose the best alternative between $X_{BestRobust}$ and the solution to the SP problem (see next section) as the final solution.

Efficiently computing the worst-case energy consumption of a route in the current solution is critical to the two-phase heuristic. Checking whether the worst-case energy consumption of vehicle k is feasible (i.e., if $w_k \leq Q$) is equivalent to checking whether the optimal value of (42)–(43) is at most Q. With the budgeted uncertainty sets U_k^2 and U_k^3, this can be done efficiently by sorting the values of $\left(\hat{a}_{ij} + \hat{b}_{ij} \cdot f_{ijk}\right)$ for the arcs in the route of vehicle k and setting the ζ_{ijk} variables in problem (42)–(43) accordingly. For example, with route-dependent uncertainty sets U_k^3, because $\hat{a}_{ij}, \hat{b}_{ij} \geq 0 \ \ \forall (i,j) \in A$, the optimal solution of problem (42)–(43) is

- $\zeta_{ijk} = 1$ for the $\lfloor \theta_0 + \theta \sum\limits_{(i,j)\in A} x_{ijk} \rfloor$ arcs with the largest $\left(\hat{a}_{ij} + \hat{b}_{ij} \cdot f_{ijk}\right)$ values in the route of vehicle k,
- $\zeta_{ijk} = \theta_0 + \theta \sum\limits_{(i,j)\in A} x_{ijk} - \lfloor \theta_0 + \theta \sum\limits_{(i,j)\in A} x_{ijk} \rfloor$ for the arc with the $\lfloor \theta_0 + \theta \sum\limits_{(i,j)\in A} x_{ijk} \rfloor + 1^{th}$ largest $\left(\hat{a}_{ij} + \hat{b}_{ij} \cdot f_{ijk}\right)$ value in the route of vehicle k,
- $\zeta_{ijk} = 0$ otherwise.

With the ellipsoidal uncertainty sets U_k^5, solving (42)–(43) can also be done efficiently since the Karush-Kuhn-Tucker optimality conditions yield the following closed formula for its optimal solution (Ilyina 2017) :

$$\zeta_{ijk} = \frac{r \cdot \sum\limits_{(l,m)\in A} \sigma_{(i,j),(l,m)} \cdot \left(\hat{a}_{lm} \cdot x_{lmk} + \hat{b}_{lm} \cdot f_{lmk}\right)}{\sqrt{\sum\limits_{(p,q)\in A} \left(\hat{a}_{pq} \cdot x_{pqk} + \hat{b}_{pq} \cdot f_{pqk}\right) \cdot \left(\sum\limits_{(l,m)\in A} \sigma_{(p,q),(l,m)} \cdot \left(\hat{a}_{lm} \cdot x_{lmk} + \hat{b}_{lm} \cdot f_{lmk}\right)\right)}} \quad \forall (i,j) \in A,$$

where $\sigma_{(i,j),(l,m)}$ is the covariance between arcs (i,j) and (l,m), or the variance in the case that $(i,j) = (l,m)$. The optimal value of (42)–(43) is then

$$\sum_{(i,j)\in A} \left(a_{ij} \cdot x_{ijk} + b_{ij} \cdot f_{ijk}\right) + r \cdot \sqrt{\sum_{(i,j)\in A} \left(\hat{a}_{ij} \cdot x_{ijk} + \hat{b}_{ij} \cdot f_{ijk}\right) \cdot \left(\sum_{(l,m)\in A} \sigma_{(i,j),(l,m)} \cdot \left(\hat{a}_{lm} \cdot x_{lmk} + \hat{b}_{lm} \cdot f_{lmk}\right)\right)}.$$

Note that the correlation between components of Z_k that are associated with arcs in the route of vehicle k and components of Z_k that are associated with arcs that are not in the route of vehicle k

does not influence the above optimal value of (42)–(43).

With the uncertainty sets U_k^4 formed by the intersection of the box uncertainty set with the ball of radius γ centered at the origin, problem (42)–(43) becomes a quadratically constrained program. We therefore use a commercial solver within the LNS phase itself to check the robustness of new routes when solving the problem with sets U_k^4. Since this renders the evaluation of the worst-case energy consumption of a route more difficult, the benefit of not forcing robustness throughout the LNS phase becomes self-evident.

Finally, we mention that we store the information regarding the robustness of new routes (i.e., their worst-case energy consumption) for future use so that the above computations only need to be performed the first time a route is encountered. The robustness of previously encountered routes can thus be checked quickly when updating $X_{BestRobust}$.

5.7 Set partitioning phase

Once the LNS phase has run for κ iterations, an SP problem is solved over the pool of routes Ω_R to find a robust solution X_{SP}. Recall that all routes in Ω_R are robust, i.e., their worst-case energy consumption is less than the battery capacity. Before solving the SP problem, a robust local search (RLS) is performed on the best known nominal solution $X_{BestNominal}$ to get solution $X_{LNS/RLS}$. The RLS is the same as the LS described in Section 5.4, except that only those moves leading to robust routes are performed, and the neighbourhoods are always searched in the same order. The RLS uses the same neighbourhoods as the LS (i.e., *2-Opt*, *2-Opt**, *Relocate*, and *Exchange*), but evaluates energy feasibility and costs with the worst-case energy consumption of routes rather than with the expected one. All new routes encountered during the RLS are then added to Ω_R, and $X_{BestRobust}$ is updated according to $X_{LNS/RLS}$.

Let binary variable y_r take value 1 if and only if energy-robust route $r \in \Omega_R$ is chosen. Let c_r be the total robust cost of route r (i.e., with energy costs computed with the route's worst-case energy consumption), and p_{ir} be a binary parameter equal to 1 if and only if customer $i \in N_0$ is in route r. The SP problem is to

$$\text{minimize} \quad \sum_{r \in \Omega_r} c_r \cdot y_r \tag{46}$$

subject to

$$\sum_{r \in \Omega_R} p_{ir} \cdot y_r = 1 \qquad i \in N_0 \tag{47}$$

$$\sum_{r \in \Omega_R} y_r \leq |K| \tag{48}$$

$$y_r \in \{0,1\} \qquad r \in \Omega_R. \tag{49}$$

Since the maximum fleet size is ensured via constraint (48) in the SP formulation, we allow solutions to contain more routes than the fleet size during the LNS phase. However, we do not consider such solutions as potential incumbents to replace $X_{BestRobust}$, i.e., we assume that $C^R(X) = \infty$ if the number of routes in solution X is larger than $|K|$. Finally, the RLS is performed once again on the best found solution X_{SP} to the SP problem. The less costly alternative between the resulting solution $X_{SP/RLS}$ and the best found robust solution $X_{BestRobust}$ over the course of the LNS phase is then chosen as the final solution.

Preliminary experiments have indicated that solving the SP problem in two phases can accelerate its resolution. Therefore, we first minimize the number of vehicles by replacing the objective (46) with $\sum_{r\in\Omega_r} y_r$. Letting m refer to the best found number of vehicles, we then solve (46)–(49) but replace the maximum fleet size $|K|$ with $\min\{m, |K|\}$.

6. Computational experiments

We have performed extensive computational experiments in order to assess the quality of our heuristic, to compare nominal solutions with robust solutions, to illustrate the trade-off between cost and risk among different robust solutions, and to investigate the influence of different parameters in the energy consumption model and in the uncertainty sets on robust solutions. To this end, we have generated test instances and solved them with the algorithms presented in the previous two sections. We have also tested our heuristic on existing benchmark instances of related problems. All algorithms were implemented in C++ and all experiments were conducted on a cluster of 27 machines, each having two Intel(R) Xeon(R) X5675 3.07 GHz processors with 96 GB of RAM running on Linux. Each machine has 12 cores, and each instance was run using a single thread. All test instances using the reformulation and the cutting plane method described in Sections 4.1 and 4.2 were solved using CPLEX 12.8 with a time limit of three hours. The SP problem within the two-phase heuristic was also solved using CPLEX 12.8. We set a time limit of T_{SP1} seconds for the vehicle minimization phase of the SP problem, and of T_{SP2} for the cost minimization phase.

The remainder of this section is organized as follows. We first provide a description of the EVRP-ECU test instances and report the values used in the experiments for the parameters of our two-phase heuristic in Section 6.1. In Sections 6.2 and 6.3 we evaluate the performance of the heuristic on small and large EVRP-ECU instances, respectively. The quality of the two-phase method is further assessed in Section 6.4 by testing it on the related capacitated vehicle routing problem (CVRP) and robust capacitated vehicle routing problem (RCVRP). In Section 6.5, we illustrate how the robust optimization methodology can be used to achieve the right balance between cost and risk. We then investigate the impact of correlation between arcs in a given route with respect to energy consumption uncertainties in Section 6.6. Finally, in Section 6.7 we analyze the impact of specific uncertain parameters in the energy consumption model described in Section 3.2 on robust solutions.

6.1 Test instances and parameter tuning

We have generated test instances by using the customer configurations from the EVRP with non-linear charging function instances presented in Montoya et al. (2017). These instances were designed to represent a geographic space of 120×120 kilometres, which is a reasonable area to cover without en route recharging, considering the average range of modern EFVs is approximately 125 kilometres (Tretvik et al. 2017). There are five instances for each of the following number n of customers: 10, 20, 40, 80, 160, and 320. The node locations are determined according to a continuous uniform distribution, a random clustered distribution, or a mixture of both. The maximum fleet size is set to $1 + n/5$. Customer demands are randomly generated between 50 and 450 kg.

The fleet vehicles are assumed to be medium-duty electric trucks equipped with 100 kWh batteries. We have used some vehicle and expected values for the energy consumption model parameters similar to those used by Goeke and Schneider (2015) and Demir et al. (2012). However, these studies do not consider auxiliary power nor dispatching costs. Asamer et al. (2016) found that an auxiliary power of approximately 450W is most frequently consumed by a passenger electric vehicle that is roughly seven times lighter than the electric trucks we consider in terms of gross vehicle weight. To err on the side of caution, we used half of this ratio, i.e., we assume an expected auxiliary power of $P = 3.5 \cdot 450 = 1575$ W when computing the expected energy consumption parameter values a_{ij} and b_{ij}. The energy and maintenance costs are based on those used by Davis and Figliozzi (2013). To determine the fixed dispatching cost c_F, we assume the same hourly salary of the drivers as in Goeke and Schneider (2015) and a nine-hour shift; c_F is thus set to \$118. The values of the cost, vehicle, and energy consumption model-related parameters are reported in Table 1.

Since we do not have a detailed road graph associated with the arcs in the VRP graphs of our instances, we estimate the arcs' expected energy consumption parameter values a_{ij} and b_{ij} by assuming that all arcs are traveled at an expected constant speed of 45 km/h, and that the road grade θ_{ij} is zero along all arcs. We initially set the maximum deviations $\hat{a}_{ij}$ and $\hat{b}_{ij}$ to 20% of the expected values a_{ij} and b_{ij}. In Section 6.7, however, we investigate the impact of using the worst-case values from Asamer et al. (2016) for certain parameters individually.

Regarding the uncertainty sets, we have conducted experiments with several of those presented in Section 3.5, while assuming they are the same for all vehicles. The route-dependent budgeted sets U_k^3 and the uncertainty sets U_k^4 formed by the intersection of the box uncertainty set with the

Table 1: Vehicle, cost and expected energy consumption model parameter values

Parameter	Value used
Battery capacity Q	100 kWh
Curb mass w	6,350 kg
Load capacity L	3,650 kg
Frontal area A	3.912 m^2
Expected air drag coefficient C_d	0.7
Expected rolling friction coefficient C_r	0.01
Expected auxiliary power demand P	1,575 W
Gravitational constant g	9.81 m/s^2
Expected air density ρ	1.2041 kg/m^3
Expected drivetrain efficiency ϕ	1.3175
Maintenance costs c_M	\$0.00016/m
Energy cost c_E	\$0.11/kWh
Fixed cost c_F	\$118

ball of radius γ centered at the origin are used to represent the case of independent components in the random vectors Z_k, with each Z_{ijk} symmetrically distributed within the interval $[-1, 1]$. The ellipsoidal uncertainty sets U_k^5 are used to represent normally distributed random vectors Z_k, each of mean zero and with covariance matrix $\sum$. Since the support of Z_k is assumed to be $[-1, 1]$, we set standard deviations to 0.333 for each Z_{ijk}, and assume a correlation of 0.9 between the arcs. The values used for θ_0, θ, γ and r in U_k^3, U_k^4 and U_k^5 are initially set so that there is a probability of at most $\beta = 0.05$ that the real energy consumption of vehicle k be larger than w_k. These values are given in Table 2. We refer the reader to Ben-Tal et al. (2009), Poss (2013) and Ilyina (2017) for more details concerning the calibration methodology, which is too extensive to discuss here.

Table 2: Uncertainty set parameter values

Uncertainty set parameter	Value used
θ_0	4.22
θ	0.22
γ	2.45
r	1.65

Finally, we conducted some preliminary experiments on a small number of instances to fine-tune the parameters of the LNS-based algorithm. **More specifically, we used one instance for each possible number of customers n with uncertainty sets U_k^3, and we tested a number of predetermined combinations of parameter values using our best judgment. We then chose the combination that performed best on the subset of six instances. The values corresponding to this combination are reported in Table 3 and were used in all experiments of the following sections.**

Table 3: Two-phase heuristic parameter values

Parameter	Value used
Bounds ($\underline{n}_c$, $\overline{n}_c$) on number of customers to remove	$(0.1 \cdot n,\, 0.4 \cdot n)$
Parameters used for randomization (λ_{WR}, λ_{WE}, λ_{WD}, λ_{Shaw}, λ_{RG}, λ_{RR})	(3, 6, 3, 6, 1.5, 1.5)
Shaw parameters (μ_1, μ_2, μ_3, μ_4)	(0.4, 0.3, 0.15, 0.1)
Threshold δ to perform LS	1.04 if $n = 320$ 1.09 if $n = 160$ 1.18 if $n = 80$ ∞ otherwise
Threshold τ to check for new robust routes during LS	0.95 if $n = 320$ 1.05 if $n = 160$ ∞ otherwise
Factor ω used to set initial temperature	1.02
Number of LNS iterations κ before SP phase	12,500
Time limit T_{SP1} for vehicle minimization phase of the SP problem	60 seconds
Time limit T_{SP2} for cost minimization phase of the SP problem	1,200 seconds if $n = 320$ 900 seconds if $n = 160$ 600 seconds if $n = 80$ 300 seconds otherwise
Maximum number D of LNS iterations without encountering a new solution before using the *Random insertion* operator	10 if $n = 320$ 20 if $n = 160$ 90 if $n = 80$ 200 if $n = 40$ 1,000 if $n = 20$ 4,000 if $n = 10$

6.2 Performance of the two-phase method on small EVRP-ECU instances

In order to get an idea of the performance of our solution method, we have used the reformulation and cutting plane methods described in Sections 4.1 and 4.2 on small instances of the EVRP-ECU with up to 20 customers. The results are reported in Tables 4, 5 and 6 for $\beta = 0.05$ and the uncertainty sets U_k^3, U_k^4 and U_k^5, respectively. Results are given for both the nominal and the robust versions of the problem for each instance. Recall that in the nominal version, it is assumed that the energy consumption parameters never deviate from their expected values. We still conduct the SP phase (and the subsequent LS) in this case since it can only improve the final solution X_{Final} returned by the procedure. However, the parameter τ is set to zero for all runs on the nominal version of the problem.

Since the route-dependent uncertainty sets are polyhedral, we apply both the reformulation approach and the cutting plane method (with cuts (45)) for the robust version of the problem when it is solved with CPLEX and those sets. Only the cutting plane method (with cuts (44)) is applied when solving the robust version with CPLEX and the other uncertainty sets. Whenever the robust version of the problem is solved with CPLEX, we report the best found solution value, the solution time, and the method that achieved it (i.e., "R" for the reformulation or "C" for the cutting plane method; ties are broken based on solution times). When the nominal version is solved with CPLEX, we only report the best found solution value and the solution time. In this case, we simply use CPLEX to solve the mixed integer linear program (MILP) resulting from having empty uncertainty sets in the model of Section 3.4. Symmetry breaking inequalities are also added to the formulation for both the robust and nominal cases. Specifically, these inequalities force vehicles with a smaller index k to be used before those with a larger one, and to leave the depot with a larger load. For the two-phase heuristic method, in both the nominal and robust cases, we report the cost of the best found solution over 10 runs, the number of vehicles m in the best found solution, as well as the average cost and average solution time in seconds over the 10 runs.

The results show that the two-phase method clearly outperforms the reformulation and the cutting plane method on each instance when the robust version of the problem is solved. When CPLEX finds an optimal solution within the three-hour time limit, our heuristic consistently finds it in much less time. When CPLEX cannot guarantee the optimality of its best found solution, our heuristic always quickly finds a solution at least as good as the one reported by CPLEX, and usually better. The same holds for the results on the nominal version of the problem when comparing the heuristic to using CPLEX on the MILP representing the nominal case.

6.3 Performance of the two-phase method on large EVRP-ECU instances

For those instances with more than 20 customers, none of the exact methods is able to find a feasible solution within the time limit for both the robust and the nominal cases. Therefore, we

Table 4: Results for the small instances with uncertainty sets U_k^3 and $\beta = 0.05$

	Nominal						Robust						
	CPLEX		LNS+SP				CPLEX			LNS+SP			
Instance	Best	t (s)	Best	m	Avg.	t (s)	Best	Method	t (s)	Best	m	Avg.	t (s)
1-10C	294.62	2445.86	294.62	2	294.62	2.02	321.23	R	2386.77	321.23	2	321.23	2.58
2-10C	316.45	2188.09	316.45	2	316.45	1.74	319.69	C	2996.37	319.69	2	319.69	2.27
3-10C	292.62	1584.54	292.62	2	292.62	3.99	321.95	R	1737.46	321.95	2	321.95	5.32
4-10C	324.55	248.64	324.55	2	324.55	2.75	328.08	C	374.06	328.08	2	328.08	3.65
5-10C	326.88	398.02	326.88	2	326.88	2.41	454.70	R	1634.42	454.70	3	454.70	3.23
1-20C	471.45	10800	470.64	3	470.64	8.14	–	C+R	10800	810.56	5	810.56	9.34
2-20C	326.09	6652.50	326.09	2	326.09	8.49	464.55	C	10800	459.95	3	460.74	9.57
3-20C	334.82	10800	334.79	2	334.79	9.68	610.04	R	10800	486.79	3	486.81	11.05
4-20C	324.75	548.67	324.75	2	324.75	10.38	450.64	R	10800	450.64	3	450.64	11.81
5-20C	463.41	10800	462.92	3	462.92	10.56	468.37	C	10800	468.37	3	468.37	12.65

Table 5: Results for the small instances with uncertainty sets U_k^4 and $\beta = 0.05$

	Nominal						Robust						
	CPLEX		LNS+SP				CPLEX			LNS+SP			
Instance	Best	t (s)	Best	m	Avg.	t (s)	Best	Method	t (s)	Best	m	Avg.	t (s)
1-10C	294.62	2445.86	294.62	2	294.62	2.02	321.24	C	3354.75	321.24	2	321.24	4.28
2-10C	316.45	2188.09	316.45	2	316.45	1.74	319.75	C	2035.69	319.75	2	319.75	3.32
3-10C	292.62	1584.54	292.62	2	292.62	3.99	321.96	C	5108.32	321.96	2	321.96	15.47
4-10C	324.55	248.64	324.55	2	324.55	2.75	328.10	C	204.48	328.10	2	328.10	6.96
5-10C	326.88	398.02	326.88	2	326.88	2.41	454.72	C	2920.70	454.72	3	454.72	4.93
1-20C	471.45	10800	470.64	3	470.64	8.14	–	C	10800	810.58	5	810.58	16.65
2-20C	326.09	6652.50	326.09	2	326.09	8.49	464.64	C	10800	459.93	3	462.72	16.61
3-20C	334.82	10800	334.79	2	334.79	9.68	609.45	C	10800	486.88	3	486.92	18.49
4-20C	324.75	548.67	324.75	2	324.75	10.38	451.14	C	10800	450.65	3	450.65	17.77
5-20C	463.41	10800	462.92	3	462.92	10.56	472.22	C	10800	468.45	3	468.45	22.02

Table 6: Results for the small instances with uncertainty sets U_k^5 and $\beta = 0.05$

	Nominal						Robust						
	CPLEX		LNS+SP				CPLEX			LNS+SP			
Instance	Best	t (s)	Best	m	Avg.	t (s)	Best	Method	t (s)	Best	m	Avg.	t (s)
1-10C	294.62	2445.86	294.62	2	294.62	2.02	295.93	C	640.76	295.93	2	295.93	2.53
2-10C	316.45	2188.09	316.45	2	316.45	1.74	318.19	C	1379.18	318.19	2	318.19	2.30
3-10C	292.62	1584.54	292.62	2	292.62	3.99	308.45	C	1210.09	308.45	2	308.45	5.36
4-10C	324.55	248.64	324.55	2	324.55	2.75	326.47	C	605.05	326.47	2	326.47	3.62
5-10C	326.88	398.02	326.88	2	326.88	2.41	333.65	C	207.45	333.65	2	333.65	3.16
1-20C	471.45	10800	470.64	3	470.64	8.14	643.58	C	10800	643.14	4	643.14	9.66
2-20C	326.09	6652.50	326.09	2	326.09	8.49	455.59	C	10800	328.13	2	328.13	9.62
3-20C	334.82	10800	334.79	2	334.79	9.68	481.60	C	10800	480.96	3	480.96	10.92
4-20C	324.75	548.67	324.75	2	324.75	10.38	449.90	C	10800	330.65	2	330.65	11.60
5-20C	463.41	10800	462.92	3	462.92	10.56	466.05	C	10800	465.28	3	465.28	12.51

only report the performance of the metaheuristic for these instances. The results are presented in Table 7 for the nominal version of the problem and for the robust version with the three types of uncertainty sets used. We once again report the cost of the best found solution over 10 runs, the number of vehicles in the best found solution, as well as the average cost and time over the 10 runs for each instance and variation. When no value is reported, this means that the algorithm was unable to find a feasible robust solution (the SP problem is infeasible and no robust solution was found during the LNS phase). It is conceivable that such instances are infeasible, i.e., there is a customer i for which there exists no route k visiting i so that $w_k \leq Q$, or routes require to be very short in order to be energy-robust, thereby requiring too many vehicles.

Despite the fact that we do not have benchmark solutions for these larger EVRP-ECU instances to evaluate the performance of the metaheuristic, the algorithm seems relatively stable when comparing the average cost over ten runs with the best run for most instances. The solution times could be considered high for some instances with 80 customers or more, but this is partly due to the fact that the set partitioning problem is sometimes not solved to optimality and therefore uses the entire time allotted to it. We mention, however, that when this occurs the optimality gap of the best found solution for the SP problem is usually relatively small (except for the 320-customer instances), and the solution is also subsequently improved through the final robust local search phase. The results with the uncertainty sets U_k^4 formed by the intersection of the box uncertainty set with the ball of radius γ centered at the origin show the importance of using sets allowing to efficiently check the robustness of routes. Recall from Section 5.6 that with uncertainty sets U_k^4, we solve a quadratically constrained program (with CPLEX) to check whether a new route is robust. This results in much larger solution times for the 320-customer instances.

It is also interesting to observe the difference between the best found solutions in nominal and robust versions of the problem. Indeed, for both the small instances results in Tables 4, 5 and 6, and the large instances results in Table 7, there are often significant differences between the values of the best found nominal and the best found robust solutions. Taking into account the energy consumption uncertainty sometimes results in using the same number of vehicles but routing them differently in order to be robust, or in using more vehicles than in the nominal case. The uncertainty sets U_k^3 and U_k^4 make the same assumptions regarding the random vectors Z_k and hence return similar solutions. The sets U_k^5, on the other hand, assume that we have much more information regarding the distribution of Z_k, and the robust solutions become less conservative than with the other two sets. There are even instances that appear to be infeasible with the other two uncertainty sets but not with sets U_k^5.

We further demonstrate the difference between nominal and robust solutions through Table 8, in which we report two measures pertaining to the best found nominal solutions. First, for each instance we report the number of routes in the best found nominal solution (out of the total in the solution) that violate constraints (17) under each uncertainty set. This indicates how often

Table 7: Results for the large instances with sets U_k^3, U_k^4 and U_k^5, and with $\beta = 0.05$

	Nominal				Uncertainty sets U_k^3				Uncertainty sets U_k^4				Uncertainty sets U_k^5			
Instance	Best	m	Avg.	t (s)	Best	m	Avg.	t (s)	Best	m	Avg.	t (s)	Best	m	Avg.	t (s)
1-40C	632.40	4	632.40	34.65	646.45	4	646.45	39.86	646.64	4	646.64	55.69	636.25	4	636.25	39.50
2-40C	999.64	6	999.64	34.56	–	–	–	–	–	–	–	–	–	–	–	–
3-40C	632.39	4	632.39	36.13	789.92	5	789.92	41.39	789.95	5	789.95	63.40	636.42	4	636.42	41.10
4-40C	473.03	3	473.03	40.75	601.91	4	601.91	45.91	601.92	4	601.92	71.35	481.63	3	481.63	48.24
5-40C	639.31	4	639.31	46.73	801.60	5	801.61	55.57	801.81	5	801.81	97.38	787.37	5	787.66	69.04
1-80C	931.18	6	931.18	219.11	941.35	6	941.35	285.57	941.28	6	941.28	399.75	938.90	6	938.90	301.20
2-80C	950.52	6	950.68	203.23	1109.76	7	1109.76	248.97	1109.80	7	1109.80	356.81	970.96	6	1052.91	261.02
3-80C	903.02	6	903.02	251.74	922.98	6	925.07	754.40	922.43	6	923.70	970.16	911.70	6	911.76	454.58
4-80C	896.95	6	896.95	233.43	904.13	6	904.13	351.53	904.01	6	904.01	587.69	901.32	6	901.32	293.16
5-80C	943.82	6	943.82	224.74	1111.80	7	1113.62	232.13	1111.71	7	1113.25	377.38	971.58	6	1008.63	403.03
1-160C	1862.32	12	1863.63	1814.32	–	–	–	–	–	–	–	–	2068.06	13	2165.07	1933.53
2-160C	1777.43	12	1778.91	2224.54	1794.86	12	1795.35	2360.19	1794.59	12	1795.07	3349.30	1786.01	12	1788.18	2264.71
3-160C	1670.80	11	1672.70	1571.13	1683.89	11	1686.06	1609.33	1683.70	11	1686.30	2504.15	1680.14	11	1683.09	1776.55
4-160C	1684.96	11	1687.88	1470.01	1872.68	12	1929.74	1431.21	1869.79	12	1890.05	2490.75	1711.48	11	1714.93	1293.94
5-160C	1729.75	11	1731.91	1267.17	–	–	–	–	–	–	–	–	1938.09	12	2064.32	1099.00
1-320C	3780.68	24	3788.77	3034.93	–	–	–	–	–	–	–	–	–	–	–	–
2-320C	3217.63	22	3221.16	5435.32	3238.64	22	3244.47	6745.20	3241.45	22	3245.28	15972.70	3229.04	22	3235.80	6093.11
3-320C	3379.40	22	3393.95	2078.88	–	–	–	–	–	–	–	–	3769.91	24	3913.29	2485.39
4-320C	3341.18	22	3384.28	6554.97	3625.48	24	3684.60	7231.61	3626.51	24	3679.36	20420.30	3458.93	23	3489.00	7813.37
5-320C	3294.02	22	3297.16	6837.66	3366.80	22	3481.77	7478.53	3363.10	22	3460.38	18871.70	3334.11	22	3354.32	7427.62

the nominal solution fails with each of the uncertainty sets. Second, for each instance we report the cost increase or decrease (in %) of the best found robust solution under each uncertainty set with respect to the best found nominal solution, when the energy costs in the nominal solution are computed with a worst-case energy consumption under the uncertainty set in question. When no value is reported for this second measure, no feasible robust solution was found for that instance and uncertainty set. As the entries show, the majority of best found nominal solutions contain one or several routes that would be infeasible in the robust version of the problem. The degree of the cost increase in the best found robust solution depends on whether more vehicles are required to ensure energy feasibility compared to the number of vehicles in the best found nominal solution. When this is the case, the robust solution is of course significantly more costly than the nominal one. On the other hand, when no additonal vehicles are required to ensure energy robustness (i.e., this can be done by rearranging the same number of routes), the cost difference between the best found robust and nominal solutions can be quite small.

6.4 Performance of the two-phase method on CVRP and RCVRP instances

In order to further validate the performance of the two-phase solution method, we have conducted a few experiments on available benchmark instances for the related CVRP and the RCVRP. The EVRP-ECU can indeed be reduced to a CVRP when the battery capacity is infinite and there are no fixed dispatching costs nor energy costs. We have therefore tested our heuristic on the well-known 14 instances (C1, ..., C14) of Christofides et al. (1979), the 20 instances (G1, ..., G20) of Golden et al. (1998), the instances of the sets A, B, and P of Augerat (1995), as well as the instances of the sets E, F, and M proposed by Christofides and Eilon (1969), Fisher (1994) and

Table 8: Feasibility of best found nominal solutions and the price of robustness

Instance	Uncertainty sets U_k^3 Number of inf. routes	Cost diff. of rob. solution (%)	Uncertainty sets U_k^4 Number of inf. routes	Cost diff. of rob. solution (%)	Uncertainty sets U_k^5 Number of inf. routes	Cost diff. of rob. solution (%)
1-10C	1/2	8.15	1/2	8.15	0/2	0.00
2-10C	0/2	0.00	0/2	0.00	0/2	0.00
3-10C	1/2	9.16	1/2	9.16	1/2	4.96
4-10C	0/2	0.00	0/2	0.00	0/2	0.00
5-10C	1/2	37.55	1/2	37.56	1/2	1.45
1-20C	2/3	70.52	2/3	70.52	2/3	35.90
2-20C	2/2	39.61	2/2	39.61	0/2	0.00
3-20C	2/2	43.82	2/2	43.85	2/2	42.72
4-20C	1/2	37.34	1/2	37.36	1/2	1.20
5-20C	1/3	0.27	1/3	0.28	0/3	0.00
1-40C	2/4	1.31	2/4	1.32	2/4	0.03
2-40C	5/6	–	4/6	–	4/6	–
3-40C	3/4	23.65	3/4	23.66	1/4	0.06
4-40C	1/3	25.97	1/3	25.97	1/3	1.22
5-40C	2/4	24.16	2/4	24.20	2/4	22.43
1-80C	1/6	0.29	1/6	0.29	1/6	0.28
2-80C	3/6	15.79	3/6	15.80	2/6	1.55
3-80C	2/6	1.41	2/6	1.36	2/6	0.46
4-80C	0/6	0.00	0/6	0.00	0/6	0.00
5-80C	2/6	16.68	2/6	16.70	1/6	2.35
1-160C	5/12	–	5/12	–	4/12	10.43
2-160C	1/12	0.18	1/12	0.18	0/12	0.01
3-160C	1/11	0.03	1/11	0.03	1/11	0.04
4-160C	4/11	10.23	4/11	10.08	4/11	1.03
5-160C	5/11	–	5/11	–	4/11	11.40
1-320C	12/24	–	12/24	–	12/24	–
2-320C	1/22	−0.07	1/22	0.03	0/22	−0.10
3-320C	8/22	–	8/22	–	8/22	10.95
4-320C	4/22	4.17	4/22	4.26	3/22	2.98
5-320C	4/22	1.43	4/22	1.34	2/22	0.72

Christofides et al. (1979), respectively. The first group contains a number of customers n ranging from 50 to 199. The second represents large-scale instances and contains between 200 and 483 customers. The number of customers in the sets A, B, P, E, F, and M varies from 12 to 199, and the number of required routes is fixed, which is handled through the SP phase and the subsequent final local search (in this case we disregard the best found number of vehicles m in the SP vehicle minimization phase, and in Algorithm 1 we assume that $C^R(X) = \infty$ if the number of routes in solution X is not equal to the required number of routes). Some instances also have maximum route durations and customer service times. We therefore adapted our heuristic to handle these extra features (feasibility with respect to route durations is enforced throughout the procedure).

The RCVRP additionally considers that the customer demands q_i are uncertain parameters. In the RCVRP, it is therefore assumed that each customer i has a certain nominal demand q_i^0 from which we expect the realized customer demand q_i to deviate according to some uncertainty set. We have tested our heuristic on the RCVRP variants of the above instances, which were introduced by Gounaris et al. (2013) and Gounaris et al. (2016). We solve these instances with the following two uncertainty sets for the customer demands (which are those used by Gounaris et al. 2013 and Gounaris et al. 2016 in their experiments):

$$U_Q^1 = \{q \in \mathbb{R}_+^n \mid \underline{q}_i \leq q_i \leq \overline{q}_i \; \forall i \in N_0, \sum_{i \in B_j} q_i \leq b_j \text{ for } j = 1, ..., J\}, \tag{50}$$

$$U_Q^2 = \{q \in \mathbb{R}_+^n \mid q = q^0 + \Gamma\xi \text{ for some } \xi \in \Xi\}, \tag{51}$$

where $\Xi = \{\xi \in \mathbb{R}^F \mid \xi \in [-e, +e], e^T\xi \in [-\beta F, +\beta F]\}, q^0 \in \mathbb{R}_+^n, \Gamma \in \mathbb{R}^{n \times F}, F \in \mathbb{N}, \beta \in [0, 1]$, and $e \in \mathbb{R}^F$ is a vector with only ones. The uncertainty set U_Q^1 states that each customer demand q_i must lie within a given interval $[\underline{q}_i, \overline{q}_i]$, and that the sum of the demands of the customers in each subset $B_j \subseteq N_0$ must be at most b_j. The uncertainty set U_Q^2 states that the demand vector q is the sum of a nominal demand vector q^0 and a disturbance $\Gamma\xi$ that depends on F independent factors, thereby allowing correlations between customer demands to be taken into account. We modified our two-phase heuristic so that routes are added to the pool of robust routes Ω_R when their worst-case total demand is at most the vehicle's load capacity L. We use propositions from Gounaris et al. (2013) to efficiently compute a route's worst-case total demand with uncertainty sets U_Q^1 and U_Q^2. We also mention that the number of required routes is fixed in all RCVRP instances (even those based on the Christofides et al. 1979 and the Golden et al. 1998 CVRP instances), and that the vehicle capacity from the associated CVRP instance is always increased by 20%. We refer the reader to Gounaris et al. (2016) for more information on the generation of the RCVRP instances.

For all parameters of the two-phase heuristic that depend on the number of customers n in Table 3, if $n \geq 320$, we use the same values as for $n = 320$; if $160 \leq n < 320$, we use the same values as for $n = 160$; etc. We set parameter τ to zero in the two-phase heuristic when solving the CVRP, as for the experiments on the nominal version of the EVRP-ECU. The detailed results are reported

Table 9: Summarized results for the CVRP and RCVRP benchmark instances

Instance set	Num. of instances	CVRP		RCVRP-U_Q^1		RCVRP-U_Q^2	
Instances of the set A (Augerat 1995)	26	Num. of new best solutions:	0	Num. of new best solutions:	1	Num. of new best solutions:	1
		Num. of matched NPO-BKSs:	0	Num. of matched NPO-BKSs:	13	Num. of matched NPO-BKSs:	9
		Num. of matched PO-BKSs:	26	Num. of matched PO-BKSs:	12	Num. of matched PO-BKSs:	16
		Avg. gap (%) when BFS>BKS:	–	Avg. gap (%) when BFS>BKS:	0.41	Avg. gap (%) when BFS>BKS:	–
Instances of the set B (Augerat 1995)	23	Num. of new best solutions:	0	Num. of new best solutions:	1	Num. of new best solutions:	0
		Num. of matched NPO-BKSs:	0	Num. of matched NPO-BKSs:	8	Num. of matched NPO-BKSs:	8
		Num. of matched PO-BKSs:	23	Num. of matched PO-BKSs:	12	Num. of matched PO-BKSs:	15
		Avg. gap (%) when BFS>BKS:	–	Avg. gap (%) when BFS>BKS:	0.53	Avg. gap (%) when BFS>BKS:	–
Instances of the set P (Augerat 1995)	24	Num. of new best solutions:	0	Num. of new best solutions:	0	Num. of new best solutions:	0
		Num. of matched NPO-BKSs:	0	Num. of matched NPO-BKSs:	14	Num. of matched NPO-BKSs:	13
		Num. of matched PO-BKSs:	24	Num. of matched PO-BKSs:	10	Num. of matched PO-BKSs:	11
		Avg. gap (%) when BFS>BKS:	–	Avg. gap (%) when BFS>BKS:	–	Avg. gap (%) when BFS>BKS:	–
Instances of the sets E, M and F (Christofides and Eilon 1969, Fisher 1994, Christofides et al. 1979)	17	Num. of new best solutions:	0	Num. of new best solutions:	1	Num. of new best solutions:	1
		Num. of matched NPO-BKSs:	0	Num. of matched NPO-BKSs:	6	Num. of matched NPO-BKSs:	8
		Num. of matched PO-BKSs:	17	Num. of matched PO-BKSs:	8	Num. of matched PO-BKSs:	7
		Avg. gap (%) when BFS>BKS:	–	Avg. gap (%) when BFS>BKS:	0.66	Avg. gap (%) when BFS>BKS:	0.81
Instances of Christofides et al. (1979)	14	Num. of new best solutions:	0	Num. of new best solutions:	7	Num. of new best solutions:	6
		Num. of matched NPO-BKSs:	4	Num. of matched NPO-BKSs:	6	Num. of matched NPO-BKSs:	7
		Num. of matched PO-BKSs:	5	Num. of matched PO-BKSs:	0	Num. of matched PO-BKSs:	0
		Avg. gap (%) when BFS>BKS:	0.39	Avg. gap (%) when BFS>BKS:	0.08	Avg. gap (%) when BFS>BKS:	0.57
Instances of Golden et al. (1998)	20	Num. of new best solutions:	0	Num. of new best solutions:	11	Num. of new best solutions:	13
		Num. of matched NPO-BKSs:	2	Num. of matched NPO-BKSs:	0	Num. of matched NPO-BKSs:	1
		Num. of matched PO-BKSs:	0	Num. of matched PO-BKSs:	0	Num. of matched PO-BKSs:	0
		Avg. gap (%) when BFS>BKS:	1.49	Num. of instances with no FS:	7	Num. of instances with no FS:	6
				Avg. gap (%) when BFS>BKS:	0.36	Avg. gap (%) when BFS>BKS:	–

in Tables 14–18 of Appendix A for the CVRP and RCVRP instances. We limit our presentation to instances for which benchmark solutions are available for both the CVRP and RCVRP (there are a few CVRP instances of sets A, E and M for which no corresponding RCVRP instances were generated by Gounaris et al. 2013). In Tables 14–18, we report the best found solution (BFS) value by our heuristic over 10 runs, the average cost and average solution time, the best known solution (BKS) value from the literature, as well as the gap (%) of the BFS by our heuristic with respect to the BKS. Solutions that are known to be optimal are marked by an asterisk in Tables 14–18.

We provide a summary of the detailed results of Appendix A in Table 9, where we distinguish between BKSs that are provenly optimal and those that are not. For each set of instances, we report the number of instances for which we identified new best solutions, the number of instances for which the BFS by our heuristic matches the non-proven optimal BKS (NPO-BKS), the number of instances for which our BFS matches the proven optimal solution (PO-BKS), and the average gap for the instances for which our BFS is worse than the BKS (whether the BKS is proven optimal or not).

Our heuristic finds the BKS for all the tested CVRP instances from sets A, B, P, E, M, and F. It also performs fairly well on the CVRP instances of Christofides et al. (1979); the BKS is found for several of these instances, and the gap is quite small when it is not. The performance of the heuristic deteriorates, however, on the larger instances of Golden et al. (1998), with a worst-case gap of 3.72% on these instances (obtained for an instance with 480 customers). Finally, the method matches or improves the BKS reported by Gounaris et al. (2016) for most RCVRP instances. The 42 new best solutions found by the heuristic are reported in boldface in Tables

14–18. There are, however, a few large-scaled RCVRP instances based on the Golden et al. (1998) CVRP instances for which the current configuration of the two-phase heuristic was unable to find a feasible robust solution over the 10 runs (i.e., no solution with only robust routes is encountered during the LNS phase, and CPLEX fails to find a feasible solution to the SP problem). These are reported in Table 9 as "Num. of instances with no FS". It is conceivable that this could be addressed (and that the method's performance as a whole on the CVRP and RCVRP instances could be further improved) by recalibrating the parameters of the algorithm. It is also possible that an adaptive layer, although not particularly useful on the EVRP-ECU instances, could also increase the heuristic's competitiveness for these problem variants. For sake of brevity we do not investigate this any further. Nonetheless, we find new best solutions for 24 of the large-scaled RCVRP instances.

6.5 The trade-off between cost and risk

A nice feature of the robust optimization methodology is that it can be used to present different alternatives to the decision maker who can then determine when and whether cost savings justify incurring extra risks. We illustrate this by solving the EVRP-ECU instances with the ellipsoidal uncertainty sets U_k^5 representing normally distributed random vectors Z_k, but we now calibrate them so that $\beta = 0.01$ instead of 0.05 as in the experiments of the previous sections. The parameter r is thus increased to 2.33. Moreover, we solve the instances using the box uncertainty sets U_k^1 as well, which is equivalent to setting $\beta = 0$ and solving a nominal version of the problem in which the expected values of the energy consumption parameters are all set to their worst-case values. The results are reported in Table 10, with the best of 10 runs presented for the new experiments. The previous best found solutions with sets U_k^5 and $\beta = 0.05$ are also reported.

As the results show, in some cases, the difference in cost between the three protection levels is rather insignificant, and one would therefore benefit from simply using the safest solution, i.e., the routing plan obtained with the box uncertainty set. However, in other cases, there is a case to be made for incurring a small risk. Indeed, for some instances, the best found solution with $\beta = 0.01$ allows interesting cost savings compared to the no-risk case of $\beta = 0$, sometimes even using fewer vehicles. The same holds when going from $\beta = 0.01$ to $\beta = 0.05$. In addition, some instances that appear to be infeasible for a smaller β become feasible by increasing the level of risk. Nevertheless, it is ultimately up to the decision maker to choose the right balance between cost and risk in the presence of uncertain parameters.

6.6 The impact of correlation in routes

In this section we take a quick look at the sensitivity of solutions to the correlation between the arcs in a given route, with respect to the deviation of their realized energy consumption parameter values

Table 10: Best found solutions with different uncertainty sets and different protection levels

	Sets $U_k^5/\beta = 0.05$		Sets $U_k^5/\beta = 0.01$		Sets $U_k^1/\beta = 0$	
Instance	Value	m	Value	m	Value	m
1-10C	295.93	2	296.47	2	321.25	2
2-10C	318.19	2	318.91	2	319.75	2
3-10C	308.45	2	315.78	2	321.97	2
4-10C	326.47	2	327.26	2	328.17	2
5-10C	333.65	2	453.75	3	454.74	3
1-20C	643.14	4	644.65	4	810.66	5
2-20C	328.13	2	456.29	3	464.25	3
3-20C	480.96	3	482.53	3	487.11	3
4-20C	330.65	2	449.96	3	450.95	3
5-20C	465.28	3	466.25	3	468.57	3
1-40C	636.25	4	637.76	4	776.44	5
2-40C	–	–	–	–	–	–
3-40C	636.42	4	646.41	4	790.29	5
4-40C	481.63	3	600.90	4	602.26	4
5-40C	787.37	5	794.31	5	805.45	5
1-80C	938.90	6	941.25	6	944.26	6
2-80C	970.96	6	1102.50	7	1112.45	7
3-80C	911.70	6	919.47	6	930.79	6
4-80C	901.32	6	903.12	6	905.27	6
5-80C	971.58	6	1101.59	7	1115.56	7
1-160C	2068.06	13	–	–	–	–
2-160C	1786.01	12	1792.05	12	1797.04	12
3-160C	1680.14	11	1683.14	11	1687.87	11
4-160C	1711.48	11	1852.28	12	1880.51	12
5-160C	1938.09	12	2374.66	15	–	–
1-320C	–	–	–	–	–	–
2-320C	3229.04	22	3239.65	22	3244.09	22
3-320C	3769.91	24	4410.57	28	–	–
4-320C	3458.93	23	3492.74	23	3626.51	24
5-320C	3334.11	22	3347.02	22	3378.22	22

Table 11: Best found solutions with sets U_k^5, $\beta = 0.01$, and different correlation levels

	No correlation		Correlation of 0.3		Correlation of 0.6		Correlation of 0.9	
Instance	Value	m	Value	m	Value	m	Value	m
1-10C	295.73	2	296.02	2	296.26	2	296.47	2
2-10C	317.61	2	318.16	2	318.56	2	318.91	2
3-10C	308.09	2	308.50	2	308.82	2	315.78	2
4-10C	326.17	2	326.61	2	326.96	2	327.26	2
5-10C	331.20	2	333.64	2	453.40	3	453.75	3
1-20C	486.00	3	643.53	4	644.12	4	644.65	4
2-20C	327.17	2	327.97	2	451.22	3	456.29	3
3-20C	475.05	3	480.93	3	481.72	3	482.53	3
4-20C	325.92	2	328.76	2	449.56	3	449.96	3
5-20C	464.51	3	465.25	3	465.80	3	466.25	3
1-40C	634.46	4	635.94	4	636.95	4	637.76	4
2-40C	–	–	–	–	–	–	–	–
3-40C	635.31	4	636.41	4	637.24	4	646.41	4
4-40C	480.36	3	481.60	3	600.39	4	600.90	4
5-40C	652.91	4	785.75	5	788.86	5	794.31	5
1-80C	936.09	6	938.36	6	939.84	6	941.25	6
2-80C	956.74	6	968.79	6	1094.47	7	1102.50	7
3-80C	908.85	6	911.39	6	913.53	6	919.47	6
4-80C	899.37	6	901.03	6	902.18	6	903.12	6
5-80C	951.67	6	971.30	6	1095.40	7	1101.59	7
1-160C	1927.93	12	2069.12	13	–	–	–	–
2-160C	1782.77	12	1785.76	12	1790.07	12	1792.05	12
3-160C	1674.56	11	1679.22	11	1681.18	11	1683.14	11
4-160C	1691.98	11	1707.40	11	1722.58	11	1852.28	12
5-160C	1880.24	12	2040.34	13	2210.65	14	2374.66	15
1-320C	–	–	–	–	–	–	–	–
2-320C	3225.95	22	3232.31	22	3233.93	22	3239.65	22
3-320C	3563.99	23	3877.77	25	4223.12	27	4410.57	28
4-320C	3423.48	23	3459.52	23	3476.16	23	3492.74	23
5-320C	3303.41	22	3323.69	22	3336.18	22	3347.02	22

from their expected ones, i.e., correlation between the components of Z_k. As briefly mentioned in Section 3.5, it is reasonable to expect driver behaviour and external conditions like the weather and road conditions to be fairly similar along the same route. We therefore investigate the impact of such dependencies on robust solutions by solving the EVRP-ECU instances with uncertainty sets U_k^5 and a protection level of $\beta = 0.01$, but by using three new correlation levels in the covariance matrix $\sum$: 0.0, 0.3 and 0.6. We report the best of 10 runs for each case in Table 11. The values in the column associated with the previously used correlation of 0.9 between the arcs are taken from Table 10. The entries suggest that the cost and number of vehicles in the best found solutions tend to increase with the degree of correlation between the components of random vectors Z_k. There is even an instance with 160 customers that appears to become infeasible when going from correlations of 0.3 to 0.6.

6.7 The impact of specific parameters in the energy consumption model

Our last computational study concerns the impact of specific parameters in the energy consumption model described in Section 3.2 on robust solutions. To this end, we modify the instances by computing the maximum deviations $\hat{a}_{ij}$ and $\hat{b}_{ij}$ from the expected values of the arcs' energy parameters with worst-case values inspired from those from Asamer et al. (2016) for the rolling friction coefficient C_r, the drivetrain efficiency ϕ, the auxiliary power P, and the product $C_d \cdot \rho$ of the air drag coefficient and air density. Since the expected auxiliary power and air drag coefficient are expected to be larger for a truck than for a small passenger vehicle as the one considered by Asamer et al. (2016), we simply use the same ratio as these authors between the worst-case and expected values to determine the worst-case associated with the expected values that we used for these parameters. Finally, we assume that speed can be up to 20% larger or smaller than the expected value of 45 km/h. Taking negative deviations from the expected value into account is only relevant for speed, since a lower speed can increase the energy consumed by auxiliaries. All worst-case and expected values are reported in Table 12.

Table 12: Nominal and worst-case energy consumption model parameter values used to assess the impact of specific parameters

Energy consumption model parameter	Expected value	Worst-case value
Air drag coefficient C_d	0.7	0.735
Rolling friction coefficient C_r	0.01	0.014
Auxiliary power demand P	1,575 W	4,431 W
Air density ρ	1.2041 kg/m^3	1.296 kg/m^3
Drivetrain efficiency ϕ	1.3175	1.4706
Speed	45 km/h	36 km/h or 54 km/h

We either change the instances by determining $\hat{a}_{ij}$ and $\hat{b}_{ij}$ with the worst-case value of only one parameter in the energy consumption model (two in the case of the product $C_d \cdot \rho$) and with the expected values of the rest, or with the worst-case values of all parameters simultaneously. All experiments were run using the ellipsoidal uncertainty sets U_k^5 with the initial correlation level of 0.9 between the components of Z_k and with a protection level of $\beta = 0.01$. The best found solutions over 10 runs are reported in Table 13 for each case, as well as for the previously solved nominal scenario with no uncertainty. For example, column "Rolling friction" reports best found solutions when $\hat{a}_{ij}$ and $\hat{b}_{ij}$ are computed with the worst-case value of C_r, and with expected values for C_d, P, ρ, ϕ and speed. Note that when traveling at constant speeds (as is the case in our EVRP-ECU instances), the worst-case energy consumption of an arc will always be attained with the smallest or largest possible speed. For an arc (i, j) of length d_{ij} traveled entirely at a constant speed of v_{ij}, let $B_{ij}(v_{ij})$ refer to the portion of the energy consumption when traveling (i, j) that is dependent on v_{ij}. Equations (1) and (2) allow expressing $B_{ij}(v_{ij})$ as

$$B_{ij}(v_{ij}) = \frac{1}{3.6 \cdot 10^6}\left(0.5\phi A C_d \rho v_{ij}^2 d_{ij} + P \cdot t_{ij}\right)$$
$$= \frac{1}{3.6 \cdot 10^6}\left(0.5\phi A C_d \rho v_{ij}^2 d_{ij} + P \cdot \frac{d_{ij}}{v_{ij}}\right),$$

which is a convex function of v_{ij} for $v_{ij} > 0$ when all other parameters are fixed. Therefore, for the cases of 1) taking the worst-case value of speed and the expected values of the other uncertain parameters, and 2) taking the worst-case values of all parameters simultaneously, we either use the minimal or maximal speed for each arc to determine $\hat{a}_{ij}$ and $\hat{b}_{ij}$ (i.e., the one that maximizes $B_{ij}(v_{ij})$ for that arc).

Taking all worst-case values simultaneously to compute $\hat{a}_{ij}$ and $\hat{b}_{ij}$ appears to render most instances infeasible (see column "All"). When this is not the case, the best found solution cost is significantly deteriorated compared to that of the nominal case. However, taking worst-case values of the parameters one at a time to compute $\hat{a}_{ij}$ and $\hat{b}_{ij}$ shows that the uncertainty of the rolling friction coefficient seems to have the largest impact on solutions, which is line with the findings of Asamer et al. (2016) in their sensitivity analysis of the energy consumption model parameters. The uncertainty surrounding the auxiliary power, the drivetrain efficiency and the travel speed also seems to have a considerable impact for a few instances, while the uncertainty of the air drag coefficient and of the air density seems to be less influential.

7. Conclusions

The goal of this study was to introduce and solve the EVRP-ECU, a practical transportation problem that can deal with the presence of uncertainties surrounding the energy consumption of EFVs without being overly conservative. We have modeled the problem within a robust optimization framework, and we have developed a two-phase metaheuristic to solve it. We have also implemented two exact robust optimization methods that can be used to solve small instances of the problem to optimality. We have adapted existing instances to generate new instances for the EVRP-ECU and we have run several numerical experiments on them with different types of uncertainty sets. We have shown that the solution method performs extremely well on small instances of the EVRP-ECU by comparing it to solutions obtained with the exact methods. Moreover, we have tested our heuristic on existing instances of the related RCVRP, and found 42 new best solutions in the process. Our computational study has also illustrated how the EVRP-ECU can help reach the right balance between cost and risk, and has shown the influence of different uncertain parameters and of their correlation with each other on robust solutions. We have also demonstrated the importance of taking energy consumption uncertainties into account by comparing nominal to robust solutions.

We believe that promising research avenues lie in the integration of additional features in the

Table 13: Best found solutions with uncertainty sets U_k^5, $\beta = 0.01$, and different energy consumption model parameters' worst-case values used to compute $\hat{a}_{ij}$ and $\hat{b}_{ij}$

	Nominal		Rolling friction		Efficiency		Air drag + density		Aux. power		Speed		All	
Instance	Value	m	Value	m	Value	m	Value	m	Value	m	Value	m	Value	m
1-10C	294.62	2	321.22	2	295.60	2	294.92	2	296.14	2	295.47	2	–	–
2-10C	316.45	2	319.67	2	317.74	2	316.85	2	318.52	2	317.61	2	–	–
3-10C	292.62	2	321.92	2	308.05	2	292.90	2	308.71	2	307.91	2	–	–
4-10C	324.55	2	328.10	2	325.98	2	325.00	2	326.86	2	325.85	2	–	–
5-10C	326.88	2	454.66	3	331.26	2	327.34	2	453.30	3	331.09	2	–	–
1-20C	470.64	3	810.50	5	485.70	3	471.94	3	643.86	4	485.46	3	–	–
2-20C	326.09	2	464.75	3	327.61	2	326.53	2	328.37	2	327.37	2	618.72	4
3-20C	334.79	2	487.02	3	478.85	3	335.27	2	481.45	3	474.97	3	–	–
4-20C	324.75	2	450.92	3	326.22	2	325.19	2	330.98	2	326.03	2	740.71	5
5-20C	462.92	3	468.46	3	464.68	3	463.47	3	465.74	3	464.50	3	–	–
1-40C	632.40	4	776.43	5	635.33	4	633.38	4	636.66	4	634.88	4	1067.69	7
2-40C	999.64	6	–	–	–	–	–	–	–	–	–	–	–	–
3-40C	632.39	4	797.54	5	635.50	4	633.18	4	636.83	4	635.04	4	–	–
4-40C	473.03	3	602.32	4	480.53	3	473.62	3	600.08	4	480.14	3	754.02	5
5-40C	639.31	4	805.68	5	657.43	4	648.99	4	788.59	5	654.53	4	–	–
1-80C	931.18	6	944.38	6	937.60	6	932.37	6	939.41	6	936.92	6	1368.92	9
2-80C	950.52	6	1112.50	7	962.75	6	954.08	6	1092.83	7	958.56	6	–	–
3-80C	903.02	6	930.12	6	909.92	6	903.97	6	913.13	6	909.33	6	–	–
4-80C	896.95	6	905.40	6	900.23	6	897.87	6	901.72	6	899.63	6	1063.97	7
5-80C	943.82	6	1115.53	7	957.66	6	945.47	6	1093.24	7	952.12	6	–	–
1-160C	1862.32	12	–	–	2044.35	13	1882.35	12	2212.61	14	1925.58	12	–	–
2-160C	1777.43	12	1797.75	12	1783.85	12	1779.29	12	1786.55	12	1783.43	12	–	–
3-160C	1670.80	11	1688.33	11	1677.96	11	1673.31	11	1680.70	11	1676.43	11	–	–
4-160C	1684.96	11	2008.80	13	1695.96	11	1688.70	11	1715.66	11	1695.12	11	–	–
5-160C	1729.75	11	–	–	1891.51	12	1738.36	11	2082.22	13	1884.05	12	–	–
1-320C	3780.68	24	–	–	–	–	4013.81	25	–	–	–	–	–	–
2-320C	3217.63	22	3246.29	22	3225.12	22	3219.12	22	3232.37	22	3226.28	22	–	–
3-320C	3379.40	22	–	–	3600.21	23	3413.40	22	4054.46	26	3567.40	23	–	–
4-320C	3341.18	22	3633.12	24	3436.38	23	3357.65	22	3481.36	23	3425.16	23	–	–
5-320C	3294.02	22	3520.89	23	3317.11	22	3297.40	22	3335.57	22	3308.58	22	–	–

problem. These may include, for example, the use of other types of uncertainty sets. One natural extension could be to incorporate en route recharging into the problem to render it applicable to EVRPs in mid- and long-haul contexts. Finally, the development of new solution methods for the EVRP-ECU would be welcome in order to have more available benchmark solutions for the larger instances.

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Appendix A. Detailed results for the CVRP and RCVRP benchmark instances

Table 14: Results for the set A of CVRP instances from Augerat (1995) and the associated RCVRP instances from Gounaris et al. (2013)

	CVRP					RCVRP-U_Q^1					RCVRP-U_Q^2				
Instance	BFS	Avg.	Time (s)	BKS	G (%)	BFS	Avg.	Time (s)	BKS	G (%)	BFS	Avg.	Time (s)	BKS	G (%)
A-n32-k05	784	784.00	18.13	784*	0.00	748	748.00	24.39	748*	0.00	748	748.00	29.25	748*	0.00
A-n33-k05	661	661.00	16.42	661*	0.00	642	642.00	19.57	642*	0.00	631	631.00	20.46	631*	0.00
A-n33-k06	742	742.00	15.30	742*	0.00	717	717.00	17.84	717*	0.00	710	710.00	18.71	710*	0.00
A-n34-k05	778	778.00	17.81	778*	0.00	715	716.50	18.87	715*	0.00	702	702.00	20.43	702*	0.00
A-n36-k05	799	799.00	26.97	799*	0.00	755	755.00	29.99	755*	0.00	766	767.80	38.75	766*	0.00
A-n37-k05	669	669.00	24.15	669*	0.00	650	650.00	31.69	650*	0.00	648	648.00	42.71	648*	0.00
A-n37-k06	949	949.00	23.50	949*	0.00	892	892.00	25.86	892*	0.00	892	892.00	28.72	892*	0.00
A-n38-k05	730	730.00	23.20	730*	0.00	704	705.60	29.54	704*	0.00	693	693.00	31.37	693*	0.00
A-n39-k05	822	822.00	27.43	822*	0.00	777	789.50	35.87	777*	0.00	772	778.50	36.31	772*	0.00
A-n39-k06	831	831.20	27.32	831*	0.00	787	787.00	34.01	787*	0.00	786	786.00	39.42	786*	0.00
A-n44-k06	937	937.00	36.45	937*	0.00	909	910.60	45.08	909	0.00	892	892.00	51.34	892*	0.00
A-n45-k06	944	945.70	36.13	944*	0.00	896	897.00	42.29	896*	0.00	891	892.20	47.15	891*	0.00
A-n46-k07	914	914.00	40.56	914*	0.00	888	888.60	50.64	888*	0.00	883	883.00	57.11	883*	0.00
A-n48-k07	1073	1073.00	49.59	1073*	0.00	1033	1033.00	93.00	1033	0.00	1033	1033.00	115.74	1033	0.00
A-n53-k07	1010	1010.00	57.52	1010*	0.00	974	974.00	100.70	974	0.00	967	967.00	147.82	967*	0.00
A-n54-k07	1167	1167.00	60.73	1167*	0.00	1106	1106.50	98.77	1106	0.00	1097	1099.80	138.11	1097	0.00
A-n55-k09	1073	1073.00	60.12	1073*	0.00	1030	1030.30	74.22	1030	0.00	1007	1007.00	74.63	1007*	0.00
A-n60-k09	1354	1354.00	107.61	1354*	0.00	1280	1280.00	105.61	1280	0.00	1264	1264.20	115.34	1264	0.00
A-n61-k09	1034	1034.00	76.40	1034*	0.00	983	983.00	113.32	983	0.00	974	974.00	105.62	974	0.00
A-n62-k08	1288	1288.20	222.46	1288*	0.00	1219	1221.00	236.04	1214	0.41	1201	1202.40	275.79	1201	0.00
A-n63-k09	1616	1616.00	103.07	1616*	0.00	1505	1505.00	163.48	1505	0.00	1498	1501.40	237.72	1498	0.00
A-n63-k10	1314	1315.50	112.22	1314*	0.00	**1236**	1237.60	94.07	1240	0.00	1222	1222.20	95.33	1222	0.00
A-n64-k09	1401	1401.50	135.76	1401*	0.00	1325	1326.00	137.09	1325	0.00	1314	1314.00	153.51	1314	0.00
A-n65-k09	1174	1175.50	77.02	1174*	0.00	1106	1106.00	111.98	1106	0.00	1094	1094.00	108.32	1094*	0.00
A-n69-k09	1159	1159.00	120.38	1159*	0.00	1109	1109.50	143.05	1109	0.00	1096	1096.00	139.88	1096	0.00
A-n80-k10	1763	1764.10	299.82	1763*	0.00	1662	1669.10	194.22	1662	0.00	**1644**	1650.90	222.99	1645	0.00

Table 15: Results for the set B of CVRP instances from Augerat (1995) and the associated RCVRP instances from Gounaris et al. (2013)

	CVRP					RCVRP-U_Q^1					RCVRP-U_Q^2				
Instance	BFS	Avg.	Time (s)	BKS	G (%)	BFS	Avg.	Time (s)	BKS	G (%)	BFS	Avg.	Time (s)	BKS	G (%)
B-n31-k05	672	672.00	15.63	672*	0.00	651	651.00	20.25	651*	0.00	651	651.00	22.71	651*	0.00
B-n34-k05	788	788.00	19.34	788*	0.00	768	768.00	20.41	768	0.00	748*	752.90	21.68	748	0.00
B-n35-k05	955	955.00	18.86	955*	0.00	883	883.00	20.81	883*	0.00	883	883.10	22.51	883*	0.00
B-n38-k06	805	805.00	33.08	805*	0.00	729	729.00	26.62	729*	0.00	729	729.00	28.86	729*	0.00
B-n39-k05	549	549.00	29.04	549*	0.00	532	535.60	51.36	532*	0.00	529	532.00	57.80	529*	0.00
B-n41-k06	829	829.00	32.76	829*	0.00	796	796.20	59.78	796*	0.00	791	791.00	72.95	791*	0.00
B-n43-k06	742	742.00	55.94	742*	0.00	681	681.00	51.22	681*	0.00	680	680.00	63.62	680*	0.00
B-n44-k07	909	909.00	48.35	909*	0.00	835	835.00	49.19	835*	0.00	835	835.00	61.49	835*	0.00
B-n45-k05	751	751.00	37.07	751*	0.00	701	701.00	66.95	701	0.00	680	682.90	67.75	680*	0.00
B-n45-k06	678	678.00	38.78	678*	0.00	660	660.00	294.60	660*	0.00	657	657.40	344.46	657*	0.00
B-n50-k07	741	741.00	48.82	741*	0.00	679	679.00	54.88	679*	0.00	699	699.00	64.72	699*	0.00
B-n50-k08	1312	1312.50	138.91	1312*	0.00	1224	1224.80	142.06	1224	0.00	1217	1217.50	312.81	1217	0.00
B-n51-k07	1032	1032.00	63.58	1032*	0.00	962	964.00	47.99	961	0.10	928	928.00	48.62	928*	0.00
B-n52-k07	747	747.00	176.35	747*	0.00	675	675.00	60.89	675*	0.00	670	670.00	65.35	670*	0.00
B-n56-k07	707	708.50	372.55	707*	0.00	623	623.00	88.31	623*	0.00	623	623.00	105.00	623*	0.00
B-n57-k07	1153	1156.50	64.56	1153*	0.00	1055	1055.00	77.57	1055	0.00	1052	1053.10	80.85	1052	0.00
B-n57-k09	1598	1598.00	82.23	1598*	0.00	1555	1557.20	179.84	1540*	0.96	1539	1540.00	101.63	1539*	0.00
B-n63-k10	1496	1496.00	91.53	1496*	0.00	1407	1407.00	95.74	1407	0.00	1405	1405.00	119.60	1405	0.00
B-n64-k09	861	861.00	83.43	861*	0.00	803	803.00	94.89	803*	0.00	803	803.00	115.02	803*	0.00
B-n66-k09	1316	1316.80	376.44	1316*	0.00	1251	1256.50	430.13	1251	0.00	1210*	1210.00	201.57	1210	0.00
B-n67-k10	1032	1032.00	119.55	1032*	0.00	1007	1007.40	137.82	1007	0.00	1001	1001.90	169.39	1001	0.00
B-n68-k09	1272	1273.20	407.20	1272*	0.00	**1205**	1214.80	428.27	1213	0.00	1191	1195.20	398.65	1191	0.00
B-n78-k10	1221	1226.60	465.87	1221*	0.00	1131	1131.60	199.20	1131	0.00	1130	1130.60	241.24	1130	0.00

Table 16: Results for the set P of CVRP instances from Augerat (1995) and the associated RCVRP instances from Gounaris et al. (2013)

	CVRP					RCVRP-U_Q^1					RCVRP-U_Q^2				
Instance	BFS	Avg.	Time (s)	BKS	G (%)	BFS	Avg.	Time (s)	BKS	G (%)	BFS	Avg.	Time (s)	BKS	G (%)
P-n16-k08	450	450.00	2.11	450*	0.00	439	439.00	3.05	439*	0.00	439	439.00	2.92	439*	0.00
P-n19-k02	212	212.00	3.68	212*	0.00	195	195.00	5.94	195*	0.00	195	195.00	7.00	195*	0.00
P-n20-k02	216	216.00	5.04	216*	0.00	208	208.00	7.20	208*	0.00	208	208.00	8.27	208*	0.00
P-n21-k02	211	211.00	6.38	211*	0.00	208	208.00	9.12	208*	0.00	208	208.00	10.78	208*	0.00
P-n22-k02	216	216.00	6.54	216*	0.00	213	213.00	9.66	213*	0.00	213	213.00	11.59	213*	0.00
P-n22-k08	603	603.00	5.38	603*	0.00	537	537.00	5.72	537*	0.00	557	557.00	5.48	557*	0.00
P-n23-k08	529	529.00	4.96	529*	0.00	504	504.00	7.84	504	0.00	503	503.00	7.37	503	0.00
P-n40-k05	458	458.00	23.46	458*	0.00	447	447.00	28.87	447*	0.00	447	447.00	33.41	447*	0.00
P-n45-k05	510	510.00	34.36	510*	0.00	501	501.30	35.76	501*	0.00	494	494.40	42.63	494*	0.00
P-n50-k07	554	554.00	42.62	554*	0.00	539	539.80	59.89	539	0.00	537	537.10	67.53	537	0.00
P-n50-k08	631	633.40	42.48	631*	0.00	592	592.00	45.14	592	0.00	588	588.00	45.70	588	0.00
P-n50-k10	696	696.00	38.75	696*	0.00	656	656.00	48.14	656	0.00	656	656.00	47.51	656	0.00
P-n51-k10	741	741.00	44.22	741*	0.00	707	707.00	52.59	707	0.00	698	698.00	53.04	698	0.00
P-n55-k07	568	568.00	61.94	568*	0.00	549	549.00	79.96	549	0.00	544	544.40	83.43	544*	0.00
P-n55-k08	588	588.00	59.38	588*	0.00	572	572.00	73.71	572	0.00	568	568.10	81.79	568	0.00
P-n55-k10	694	694.00	58.61	694*	0.00	670	670.00	62.56	670	0.00	657	657.00	61.14	657	0.00
P-n55-k15	989	989.00	45.24	989*	0.00	889	889.00	54.61	889	0.00	877	877.00	51.38	877	0.00
P-n60-k10	744	744.00	62.99	744*	0.00	712	712.00	71.52	712	0.00	705	705.00	73.46	705	0.00
P-n60-k15	968	968.00	58.72	968*	0.00	931	931.00	74.77	931	0.00	916	916.00	72.09	916	0.00
P-n65-k10	792	792.00	83.20	792*	0.00	765	765.20	85.78	765	0.00	761	761.50	90.67	761	0.00
P-n70-k10	827	827.00	104.02	827*	0.00	785	785.10	134.14	797	0.00	783	783.70	132.73	783	0.00
P-n76-k04	593	593.70	147.57	593*	0.00	590	590.00	248.04	590*	0.00	590	590.00	290.54	590*	0.00
P-n76-k05	627	627.30	136.09	627*	0.00	616	616.00	223.48	616	0.00	615	615.00	263.13	615	0.00
P-n101-k04	681	681.00	284.68	681*	0.00	673	673.00	240.19	673*	0.00	673	673.00	274.40	673*	0.00

Table 17: Results for the sets E, M, and F of CVRP instances and the associated RCVRP instances from Gounaris et al. (2013)

	CVRP					RCVRP-U_Q^1					RCVRP-U_Q^2				
Instance	BFS	Avg.	Time (s)	BKS	G (%)	BFS	Avg.	Time (s)	BKS	G (%)	BFS	Avg.	Time (s)	BKS	G (%)
E-n22-k04	375	375.00	5.66	375*	0.00	373	373.00	8.45	373*	0.00	373	373.00	8.46	373*	0.00
E-n23-k03	569	569.00	11.42	569*	0.00	563	563.00	16.72	563*	0.00	544	544.00	16.75	544*	0.00
E-n30-k03	534	534.00	16.43	534*	0.00	475	475.00	18.79	475*	0.00	492	492.00	19.06	492*	0.00
E-n33-k04	835	835.00	22.22	835*	0.00	814	814.00	44.44	814*	0.00	814	814.00	38.15	814*	0.00
E-n51-k05	521	521.00	46.95	521*	0.00	516	516.00	70.48	516*	0.00	516	516.00	83.10	516*	0.00
E-n76-k07	682	682.00	168.33	682*	0.00	661	661.00	226.48	661	0.00	661	661.00	236.24	661	0.00
E-n76-k08	735	735.50	143.76	735*	0.00	709	709.70	173.48	709	0.00	700	700.00	185.40	700	0.00
E-n76-k10	830	830.20	137.28	830*	0.00	796	796.10	165.91	796	0.00	782	782.00	170.14	782	0.00
E-n76-k14	1021	1021.00	128.38	1021*	0.00	952	952.00	132.49	952	0.00	952	952.00	136.84	952	0.00
E-n101-k08	815	816.20	296.91	815*	0.00	789	789.30	229.77	789	0.00	783	784.50	213.22	783	0.00
E-n101-k14	1067	1068.70	385.58	1067*	0.00	1011	1011.10	246.24	1011	0.00	1009	1009.00	234.23	1009	0.00
F-n45-k04	724	724.40	58.20	724*	0.00	718	718.50	73.17	718*	0.00	714	714.80	77.75	714*	0.00
F-n72-k04	237	237.00	180.88	237*	0.00	232	232.00	205.54	232*	0.00	232	232.00	210.91	232*	0.00
F-n135-k07	1162	1162.00	464.69	1162*	0.00	1136	1144.50	696.67	1129	0.62	1086	1087.20	440.14	1086	0.00
M-n101-k10	820	820.00	113.01	820*	0.00	809	809.10	186.06	809*	0.00	804	804.80	186.25	804*	0.00
M-n121-k07	1034	1043.90	147.03	1034*	0.00	1001	1027.00	862.91	994	0.70	990	994.00	550.74	982	0.81
M-n151-k12	1015	1024.00	983.11	1015*	0.00	**989**	994.10	1077.24	993	0.00	**971**	982.90	907.54	986	0.00

Table 18: Results for the Christofides et al. (1979) and Golden et al. (1998) CVRP instances and the associated RCVRP instances from Gounaris et al. (2016)

		CVRP					RCVRP-U_Q^1					RCVRP-U_Q^2				
Instance	n	BFS	Avg.	Time (s)	BKS	G (%)	BFS	Avg.	Time (s)	BKS	G (%)	BFS	Avg.	Time (s)	BKS	G (%)
C1	50	524.61	524.61	43.65	524.61*	0.00	519.43	519.43	71.87	519.43	0.00	519.43	519.43	94.84	519.43	0.00
C2	75	835.26	835.26	152.82	835.26*	0.00	**799.05**	799.42	205.03	807.15	0.00	785.86	785.86	192.69	785.86	0.00
C3	100	826.14	826.77	322.33	826.14*	0.00	**801.29**	802.35	274.89	803.33	0.00	794.52	796.23	232.52	794.52	0.00
C4	150	1030.83	1032.22	1368.24	1028.42*	0.23	**997.33**	1008.01	1291.08	1012.80	0.00	**982.36**	987.38	880.97	1006.33	0.00
C5	199	1295.99	1311.80	1575.52	1291.29*	0.36	**1251.29**	1262.35	1305.09	1254.38	0.00	**1236.42**	1243.71	1245.61	1243.27	0.00
C6	50	555.43	555.43	48.04	555.43	0.00	555.43	555.43	70.79	555.43	0.00	555.43	555.43	66.87	555.43	0.00
C7	75	909.68	909.68	126.95	909.68	0.00	902.01	902.01	167.77	902.01	0.00	**900.12**	901.02	182.46	901.40	0.00
C8	100	865.94	865.94	447.79	865.94	0.00	865.50	865.50	914.11	865.50	0.00	865.50	865.50	912.79	865.50	0.00
C9	150	1163.85	1165.81	1549.10	1162.55	0.11	**1161.85**	1164.51	1671.63	1167.06	0.00	**1163.48**	1165.74	1651.34	1163.81	0.00
C10	199	1408.23	1412.73	2728.72	1395.85	0.88	**1389.07**	1397.92	3138.75	1412.10	0.00	**1387.00**	1397.01	2853.88	1410.65	0.00
C11	120	1042.12	1047.95	188.04	1042.11*	0.00	1005.92	1007.23	852.08	1005.10	0.08	1000.35	1002.73	782.37	994.63	0.57
C12	100	819.56	819.56	124.34	819.56*	0.00	808.90	809.45	218.83	808.90	0.00	804.08	804.19	218.13	804.08	0.00
C13	120	1546.76	1555.54	1315.67	1541.14	0.36	**1545.96**	1559.92	1383.64	1547.06	0.00	**1542.86**	1556.15	1250.93	1544.90	0.00
C14	100	866.37	866.37	381.22	866.37	0.00	847.43	847.46	455.82	847.43	0.00	835.11	835.11	338.51	835.11	0.00
G1	240	5644.44	5669.78	2860.50	5623.47	0.37	**5618.59**	5634.48	3291.12	5694.68	0.00	**5636.22**	5637.04	3220.34	5698.06	0.00
G2	320	8447.92	8452.06	3315.92	8404.61	0.51	**8447.92**	8450.64	3638.16	8557.12	0.00	**8447.92**	8452.56	3779.75	8544.31	0.00
G3	400	11036.22	11051.80	5578.05	11036.22	0.00	**11036.22**	11047.30	5279.53	11362.36	0.00	**11036.22**	11072.00	5581.29	11423.06	0.00
G4	480	13624.53	13648.90	6698.06	13592.88	0.23	**13624.52**	13625.60	7729.72	14134.17	0.00	**13624.53**	13630.60	7774.88	13975.76	0.00
G5	200	6460.98	6460.98	965.03	6460.98	0.00	**6460.98**	6460.98	1083.11	6466.68	0.00	6460.98	6462.12	1311.87	6460.98	0.00
G6	280	8412.90	8412.95	2767.71	8404.26	0.10	**8412.90**	8412.99	3306.22	8414.28	0.00	**8412.67**	8412.88	3646.73	8415.21	0.00
G7	360	10195.59	10195.59	3901.31	10102.68	0.91	**10195.59**	10195.59	4202.26	10266.87	0.00	**10195.59**	10197.2	4054.57	10203.57	0.00
G8	440	11828.78	11835.70	8370.14	11635.34	1.64	**11744.95**	11840.40	7155.05	12078.23	0.00	**11727.83**	11817.9	7410.79	12074.27	0.00
G9	255	588.58	590.50	1831.82	579.71	1.51	572.63	575.79	1098.56	570.63	0.35	**557.86**	567.99	1334.58	562.65	0.00
G10	323	752.03	755.92	1875.66	736.26	2.10	–	–	–	736.41	–	–	–	–	724.61	–
G11	399	930.20	940.33	2183.79	912.84	1.87	–	–	–	925.88	–	–	–	–	912.17	–
G12	483	1138.07	1144.33	2707.28	1102.69	3.11	–	–	–	1181.15	–	–	–	–	1114.85	–
G13	252	862.26	871.62	1727.59	857.19	0.59	**830.95**	837.80	907.50	844.05	0.00	**825.21**	830.72	1038.67	838.49	0.00
G14	320	1101.57	1113.68	1638.26	1080.55*	1.91	**1079.85**	1089.67	1084.33	1080.59	0.00	**1060.31**	1074.00	638.75	1070.32	0.00
G15	396	1370.82	1380.98	1929.40	1337.92	2.40	1345.99	1362.28	1594.79	1341.21	0.36	**1312.29**	1334.07	1216.22	1331.96	0.00
G16	480	1674.86	1684.75	2253.96	1612.50	3.72	–	–	–	1641.95	–	**1600.30**	1611.8	2340.29	1629.12	0.00
G17	240	708.68	711.38	2076.74	707.76*	0.13	**705.89**	708.79	1747.90	713.68	0.00	**690.60**	692.90	1226.74	701.42	0.00
G18	300	1013.33	1017.71	3032.46	995.13*	1.80	–	–	–	1012.90	–	–	–	–	991.54	–
G19	360	1385.04	1395.61	2056.09	1365.60*	1.40	–	–	–	1389.32	–	–	–	–	1369.57	–
G20	420	1860.91	1902.05	2026.15	1818.32	2.29	–	–	–	1860.84	–	–	–	–	1827.02	–

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